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AUTOMATIC COMPUTATION OF ATMOSPHERIC DENSITIES

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1730 Cambridge Street
Cambridge, Massachusetts 02138

Contract No. F19628-68-C-0032

Project No. 8605,6690
Task No. 860510,669005
Work Unit No. 86051001,66900501

FINAL REPORT

Period covered: September 1967 - September 1969
Date of report: November 1969

Contract Monitor
Rosalie Schweinfurth
Aeronomy Laboratory

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Prepared for

AIR FORCE CAMBRIDGE RESEARCH LABORATORIES
OFFICE OF AEROSPACE RESEARCH
UNITED STATES AIR FORCE
BEDFORD, MASSACHUSETTS 01730

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Abstract

The Cambridge Atmospheric Density Numerical Integration Program (CADNIP) is a completely automatic computer program capable of determining atmospheric densities from an analysis of satellite observations. The adopted approach consists of a numerical integration procedure combined with a differential correction scheme where discrepancies between computed and observed satellite position and velocity are reconciled by adjusting the assumed atmospheric model, thereby yielding corrected or refined density data.

This report documents the latest version of the program which includes significant refinements and improvements incorporated into CADNIP under this contract.

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Section 1

Introduction

The IBM Cambridge Advanced Systems Department has been actively engaged in the design and development of automatic computer programs capable of determining atmospheric densities from an analysis of satellite observations. This interest (which has been continuous over the past ten years) has resulted in two totally different computational approaches to the problem. For relatively high satellites, where the effects of atmospheric drag are small, a successful method[1] developed by IBM consists of analyzing the orbital elements of the satellite over long periods of time. In particular, a precise knowledge of the rate of change of the period may be used for density determination.

For low satellites, where the effects of atmospheric drag are considerable, an alternate approach has been developed by IBM. This method consists of a numerical integration scheme combined with a differential correction procedure where the discrepancies between computed and observed satellite position and velocity are reconciled by adjusting the assumed atmospheric model, thereby yielding corrected or refined density data. An important advantage of this technique is its applicability to satellites entering the decay stage. This method[2] has also been successfully incorporated by IBM into completely automatic computer programs capable of computing atmospheric densities from an analysis of optical or electronic observations of a satellite.

The research effort reported upon herein consisted primarily of an attempt to improve the techniques developed under the latter method. These refinements were aimed at increasing the precision of the computed results, decreasing the required computer time, and incorporating additional flexibility and utility into the program.

As a result, an improved version of the Cambridge Atmospheric Density Numerical Integration Program (CADNIP) was obtained and is documented in this Final Report. Special attention should be given to Sections 5, and 6 along with Appendix B which emphasize the aspects of the research effort performed under this contract.

Section 2

Procedures Summary

This section is devoted to a brief description of the procedures used in the program.

Given a set of satellite observations covering a minimum time span, it is desired to deduce from these observations the density of the atmosphere in the region of space covered by the satellite. The procedure requires an initial estimate of the vehicle's orbital parameters along with an estimate of the nighttime exospheric temperature at some time.

The given orbital elements are converted to position and velocity.[3] The numerical integration scheme is then called for tabulating computed position and velocity at each of the observation times. A standard differential correction technique is then used to reconcile discrepancies between computed and observed position and velocity. The differential correction scheme actually corrects seven quantities; the six elements of position and velocity and the assumed atmospheric model. Finally, the corrected position and velocity are converted back to orbital elements for output purposes.[4]

An additional feature of the program consists of a preliminary adjustment routine which attempts to improve initial estimates of certain parameters most likely to be in error. This feature effectively supplies the differential correction routine with more accurate starting conditions which tend to increase the probability of convergence and also to speed up the procedure.

Upon completing the processing of one set of observations, the program proceeds to the next set of observations, with the results of the previous set being used as the initial conditions for the current epoch.

Section 3

Mathematical Considerations

This section summarizes briefly the major mathematical techniques incorporated into the program.

3.1 Numerical Integration

The perturbative forces considered for this study are the earth's gravitational field and air drag.

Geopotential Representation

The earth's geopotential is represented by

$$u = \frac{1}{r} \left[1 + \sum_{n=2}^{\infty} \frac{1}{r^n} \sum_{m=0}^{n} \left(C_n^m \cos m\lambda + S_n^m \sin m\lambda \right) P_n^m \{\sin \phi'\} \right]$$

where:

r = radial distance of the satellite from the center of the earth

λ = subsatellite longitude

ϕ' = subsatellite geocentric latitude

C_n^m, S_n^m are the earth's zonal, tesseral, and sectorial harmonics

P_n^m $\{\sin \phi'\}$ are the associated Legendre functions defined by

$$P_n^m \{x\} = (1 - x^2)^{m/2} \frac{d^m}{dx^m} P_n(x)$$

In practice, the program is capable of utilizing all zonal, tesseral, and sectorial terms through (20, 20).

Drag Considerations

The drag effect is computed from a double interpolation of Jacchia's model atmosphere [5] which gives density as a function of height and exospheric temperature. The table contains values for heights ranging between 120 and 1000 kilometers and temperatures between 2400° K and 600° K. For heights above 1000 kilometers, an exponential extrapolation is used.

The actual density calculation proceeds as follows.

The exospheric temperature (T) is computed from Jacchia's model for the diurnal variation,[6]

$$T = T_N \left\{ 1 + R \left[\sin^m \theta + (\cos^m \eta - \sin^m \theta) \cos^n \frac{\tau}{2} \right] \right\}$$

where T_N = nightime temperature

$\eta = \frac{1}{2} (\delta - \delta_{\odot})$

$\theta = \frac{1}{2} (\delta + \delta_{\odot})$

$\tau = (\alpha - \alpha_{\odot}) + \beta + p \sin (\alpha - \alpha_{\odot} + \gamma)$

where α, δ = right ascension, declination of the satellite

$\alpha_{\odot}, \delta_{\odot}$ = right ascension, declination of the sun

$R, m, n, \beta, p, \gamma$ are constants determined by Jacchia from drag studies using the Nicolet II model.

The height of the satellite is computed by subtracting the radius of the earth at the geocentric latitude of the subsatellite point from the geocentric distance of the satellite. The radius of the earth is approximated by

$$r = a_e - f \sin^2 \delta$$

where δ = geocentric latitude

f = 21.382 kilometers (difference between equatorial and polar radii of earth)

a_e = 6378.165 kilometers (equatorial radius of earth)

Two dimensional quadratic interpolation in the table is then used to achieve the desired density.

Integration Algorithm

The integration algorithm used in the program is a sixth order predictor - corrector scheme which is initiated by a classical fourth order Runge - Kutta method. The algorithm is documented separately in Appendix B of this report.

3.2 Differential Orbit Correction

Each observation to be processed gives rise to 2 or 3 (depending upon the number of measured quantities) equations of condition of the form[3]:

$$\underset{3\times 1}{\begin{pmatrix} \Delta \rho \\ \Delta \alpha \\ \Delta \delta \end{pmatrix}} = \underset{3\times 3}{\begin{pmatrix} \cos\delta\cos\alpha & \cos\delta\sin\alpha & \sin\delta \\ \dfrac{-\sin\alpha}{\rho} & \dfrac{\cos\alpha}{\rho} & 0 \\ \dfrac{-\sin\delta\cos\alpha}{\rho} & -\dfrac{\sin\delta\sin\alpha}{\rho} & \dfrac{\cos\delta}{\rho} \end{pmatrix}} \underset{3\times 7}{\begin{pmatrix} \dfrac{\partial x}{\partial x_o} & \dfrac{\partial x}{\partial y_o} & \cdots & \dfrac{\partial x}{\partial A} \\ \dfrac{\partial y}{\partial x_o} & \dfrac{\partial y}{\partial y_o} & \cdots & \dfrac{\partial y}{\partial A} \\ \dfrac{\partial z}{\partial x_o} & \dfrac{\partial z}{\partial y_o} & \cdots & \dfrac{\partial z}{\partial A} \end{pmatrix}} \underset{7\times 1}{\begin{pmatrix} dx_o \\ dy_o \\ dz_o \\ d\dot{x}_o \\ d\dot{y}_o \\ d\dot{z}_o \\ dA \end{pmatrix}}$$

where

$\Delta\rho$, $\Delta\alpha$, $\Delta\delta$ are the differences in range, right ascension, and declination between computed and observed position.

dx_o, dy_o, dz_o, $d\dot{x}_o$, $d\dot{y}_o$, $d\dot{z}_o$, dA are the desired corrections to the initial position, velocity, and atmospheric model in order to minimize $\Delta\rho$, $\Delta\alpha$, and $\Delta\delta$.

The 3 x 7 matrix gives the partial derivatives of present position with respect to initial position, velocity, and density. These derivatives are computed numerically.

The 3 x 3 matrix is for geometrical purposes.

When all observations have been processed, the resultant system of equations is solved in the least squares sense, thereby yielding the desired corrections. Corrections to position and velocity are added directly to initial position and velocity, while the correction to the atmospheric model is affected by modifying each interpolated value retrieved from the model. The entire procedure is iterated until convergence is achieved.

3.3 Preliminary Adjustments

Prior to the differential correction procedure, a preliminary adjustment is made to those parameters most likely to contain significant errors, namely, mean anomaly, semi-major axis, and the atmospheric model. Given an initial estimate of the orbital parameters of the vehicle, the numerical integration routine is called to generate computed position and velocity at each of the observation times. Differences between computed and observed position and velocity are transformed into mean anomaly residuals[3], which are then fitted with a second degree polynomial. The constant term is used as a correction to the mean anomaly, the coefficient of the linear term is used as a correction to the semi-major axis, and the coefficient of the quadratic term is used to correct the atmospheric model. (As in the differential correction procedure, the correction to the model is affected by modifying each interpolated value retrieved from the model.)

Section 4

Program Construction

The program contains 46 routines each of which is labeled and serialized in columns 73-80. All coding is in FORTRAN IV. The name and a brief description of each routine is shown in Table 4 - 1.

Table 4-1

Routines in the Cambridge Atmospheric Density Numerical Integration Program.

Name	Function
DRIVE	Activates atmospheric density processing routines.
ADMON	Main control routine.
RDPRCD	Reads and prints data cards.
STNEQX	Reads station and equinox data.
OBSET	Groups observations into sets.
PREPOB	Performs preliminary calculations on observations.
SAO	Reads Smithsonian format observations.
SPTRK1	Reads AFCRL format observations.
SPTRK2	Reads SPADATS format observations.
IPMAD	Performs preliminary adjustment of mean anomaly, semi-major axis, and atmospheric model.
POLYLS	Quadratic curve fitting routine.
DOC	Differential correction supervisor.
PRTRES	Prints residuals after differential correction.
MARESD	Computes mean anomaly residuals.
MATIV	Matrix inversion routine.
EQCDT	Forms equations of condition.
GEOMTR	Computes geometry matrix.
PTLDRV	Computes partial derivatives.
INTCRL	Control routine for numerical integration.
RUNKUT	Runge-Kutta integration routine.
ABM	Predictor-corrector integration routine.
RHSEQM	Computes right hand sides of equations of motion.
MOON	Computes lunar position (inactive at present).

Table 4-1 (continued)

Name	Function
GEOPT	Evaluates geopotential contribution.
LGNDR	Computes Legendre polynomials and associated functions.
MODEL	Chooses proper atmospheric model.
DENS	Computes density from Jacchia's model atmosphere.
DRAG	Computes density from drag data (inactive at present).
SOL	Computes solar position.
OSCTMN	Converts position and velocity to mean elements.
PVTELM	Converts position and velocity to osculating elements.
MNTOSC	Converts mean elements to position and velocity.
ELMTPV	Converts osculating elements to position and velocity.
SHPRDC	Computes short periodic perturbations.
XYZADR	Converts x, y, z to α, δ, ρ
ADTAE	Converts α, δ to azimuth and elevation
TDIFR	Computes difference between two times.
CLNTJD	Converts calendar date to modified Julian Date.
TINCR	Increments a given time.
YRMNDY	Computes date from day of year.
NDINYR	Computes number of days in a year.
NDTOD	Computes day of year from date.
TDCONV	Converts a date to output format.
KEPLER	Solves Kepler's equation.
ANGLE1	Reduces an angle to the interval ($-\pi, +\pi$).
ANGLE2	Reduces an angle to the interval ($0, 2\pi$).

Section 5

Summary of Investigations and Accomplishments

This section summarizes the investigations performed and the accomplishments achieved during this contract.

5.1 Revised Numerical Integration Algorithm

A revised numerical integration algorithm consisting of a sixth order predictor-corrector method which is started by a classical fourth order Runge-Kutta scheme was incorporated into the program resulting in a considerable saving of computer time. See Appendix B for a complete description of this method.

5.2 Elimination of Numerical Integration Re-Starts

The incorporation of the predictor-corrector method did not attain the expected reduction in computer time. It was soon determined that this was due to observational data in the time interval being considered. Each observation being processed required a re-start of the predictor-corrector method, thereby considerably reducing its effectiveness. Consequently, the logic of the numerical integration procedure was revised so as to eliminate the Runge-Kutta re-starts at each observation. This modification reduced the running time to its expected level.

5.3 Elimination of High Order Terms from Preliminary Calculations

Additional logic was incorporated into the program which allows the user to eliminate high order zonal, tesseral, and sectorial harmonics for certain preliminary calculations not requiring extreme accuracy. Specifically, a lower order geopotential model is now used for the preliminary adjustment procedure (see Section 3.3) and for early iterations in the differential correction procedure, resulting in a further reduction in running time.

5.4 Analytic Representation of the Geopotential

An attempt was made to replace the existing formulation for estimating the effects of the geopotential on the motion of a satellite by an analytic scheme. The successful completion of this task would result in a significant reduction in the running time of the program. The results of this investigation are presented in Section 6.

5.5 Calculation of Perigee Data

The accuracy of the program was improved by replacing the calculation of the final perigee time and height with a more appropriate scheme. Originally, the final perigee time and height were obtained from the final corrected orbital elements by setting the mean anomaly equal to zero and by using the keplerian formula for the period for computing the time of perigee. The procedure used now actually integrates from the time of the differential correction to perigee time. Consequently, the effects of air drag and the geopotential are now taken into account. The perigee time is computed by an iterative approach.

5.6 Recoding of Selected Portions of the Program

Selected portions of the program, including the entire geopotential as well as various utility routines used in the conversion from (to) orbital elements to (from) position and velocity, were recoded in double precision. This change yielded a more accurate and reliable procedure.

5.7 Inclusion of Drag Model

As an alternative to using the Jacchia density model, a drag model was incorporated into the program, allowing the user to supply accelerometer type data as a function of time. Thus far no input data of this form has been made available, so that the formulation has not been verified.

5.8 Modification to Output Format

The output format of the program was modifed significantly, yielding more 'readable' results. In addition, a complete listing of the parameter table (as modified by the user) is included in the output. This listing eliminates any ambiguities with regard to initial conditions.

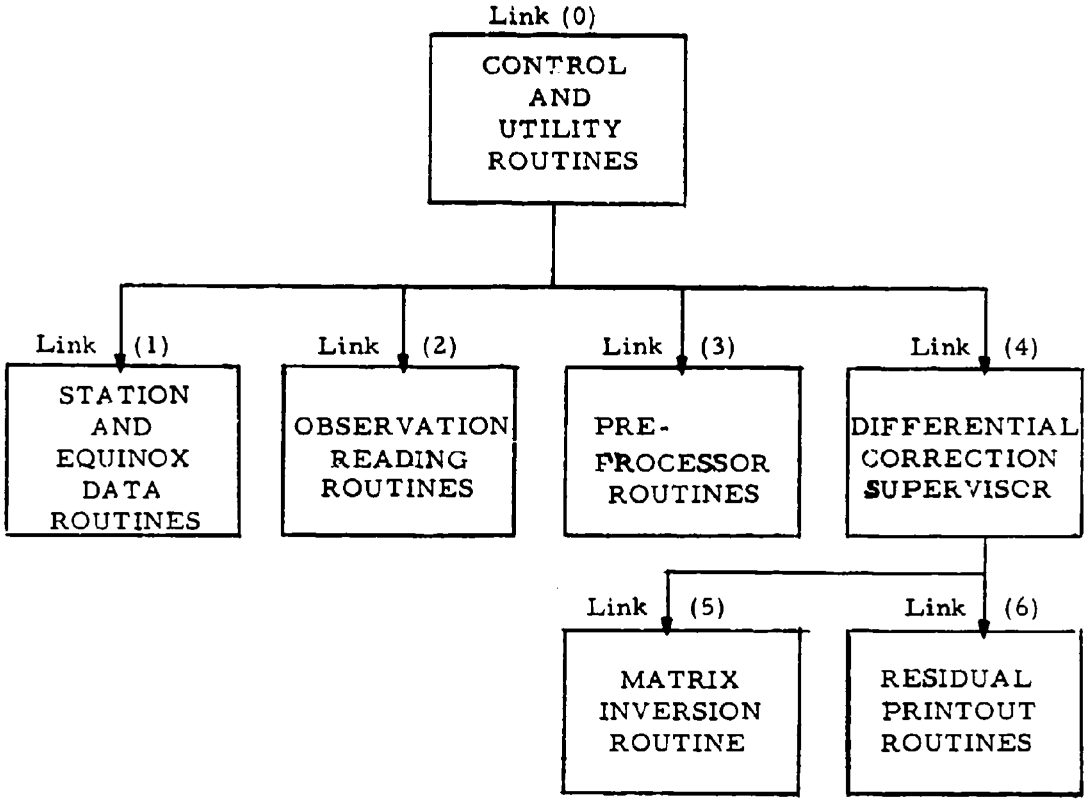
5.9 Overlaying of Program

A significant amount of core storage was made available by overlaying the CADNIP program as shown in figure 5-1.

Link 0 is the main link and resides in core storage at all times. Links 1 through 4 normally reside on a system utility device and are read into a reserved area of core storage when their execution is required. Similarly links 5 and 6 share a common area of core storage. As a result approximately 4700 decimal locations have been saved. These locations are used for additional observation buffer space, allowing the user to process up to 125 observations in one run.

Figure 5-1

Structure of Overlayed Version of CADNIP



Section 6

Analytic Representation for Geopotential

In the present version of the CADNIP Program, the variable orbital parameters are linked to the observations by means of numerical integration of the rectilinear equations of motion. Such a procedure has some very important advantages. Perturbations due to air drag, which are quite difficult to treat analytically, are handled easily with numerical integration. Numerical integration also takes interaction and higher-order effects into account automatically. It is extremely time consuming, however, and the use of CADNIP in production work results in the expenditure of large amounts of computer time.

Several refinements have been introduced into the CADNIP program (see Section 5) in order to reduce its running time. The basic premise of numerical integration has never been altered, however. Simultaneous and detailed treatment of the major atmospheric and gravitational effects is an essential requirement of the program. Yet, the possibility exists that the smaller terms in the gravitational potential could be treated by analytic methods. Since CADNIP typically utilizes a fairly complete geopotential model, a considerable amount of computer time is required to evaluate the right hand sides of the equations of motion which must be done at each step of the integration. In analytic form, it would, of course be necessary to evaluate the perturbations due to these many small terms only at the times of the observations.

Considerable time and effort has gone into the study of the possibility of an analytical representation of the higher order terms of the geopotential in CADNIP. This work has centered around a program developed to compare the perturbations resulting from the numerical integration scheme used in CADNIP with those resulting from a particular analytic development. This program determines the perturbations under numerical integration by integrating over a specified time interval with a specified set of (low-order) harmonic coefficients, integrating a second time using the original set plus any specified set of higher-order coefficients, and then taking the difference between the two results at regular intervals. Differences in the osculating Keplerian elements as well as differences in position are evaluated and compared with those computed by routines based on the analytic development of Kaula.[7,8]

Kaula's theory has been successfully applied by several groups working with earth satellite observations. It is, for example, used in the DOI program of the Smithsonian Astrophysical Observatory to compute the effects of the tesseral (and sectorial) harmonics during orbit determinations from precise photographic observations. The theory will not be described in complete detail here. A brief sketch of the development will be given, however.

The earth's potential field is usually described by a series of spherical harmonics in the form:

$$V = \frac{\mu}{r}\{1 + \sum_{n=2}^{\infty} \sum_{m=0}^{n} \frac{1}{r^n} J_{nm} P_{nm}(\sin\phi)\cos m(\lambda - \lambda_{nm})\}$$

or:

$$V = \frac{\mu}{r}\{1 + \sum_{n=2}^{\infty} \sum_{m=0}^{n} (\frac{a_e}{r})^n P_{nm}(\sin\phi)(C_{nm}\cos m\lambda + S_{nm}\sin m\lambda)\}$$

where r is the radial distance from the center of mass, ϕ the latitude and λ the (east) longitude; the J_{nm}, λ_{nm}, or C_{nm}, S_{nm} are numerical coefficients; and the $P_{nm}(\sin\phi)$ are the Legendre associated functions of the first kind defined by:

$$P_{nm}(\sin\phi) = \cos^m\phi \frac{d^m P_n(\sin\phi)}{d(\sin\phi)^m}$$

Kaula transforms the potential from spherical coordinates to Keplerian elements (ω, Ω, i, e, M, a) with the result that, for a particular term of the potential:

$$V_{nm} = \frac{\mu}{a}(\frac{a_e}{a})^n \sum_{p=0}^{n} F_{nmp}(i) \sum_{q=-\infty}^{\infty} G_{npq}(e) S_{nmpq}(\omega, M, \Omega, \theta)$$

where θ is the Greenwich Sidereal Time,

$$S_{nmpq} = \begin{bmatrix} C_{nm} \\ \\ -S_{nm} \end{bmatrix}_{\substack{n-m \text{ even} \\ n-m \text{ odd}}} \cos \left[(n-2p)\omega + (n-2p+q)M + m(\Omega - \theta)\right]$$

$$+ \begin{bmatrix} S_{nm} \\ \\ C_{nm} \end{bmatrix}_{\substack{n-m \text{ even} \\ n-m \text{ odd}}} \sin \left[(n-2p)\omega + (n-2p+q)M + m(\Omega - \theta)\right]$$

and the F(i) and G(e) are rather complicated functions which are, however, quite familiar in classical celestial mechanics.

With the potential in this form, the Lagrangian equations of motion can be integrated analytically quite easily if it is assumed that the only variations on the right hand sides are linear variations in the argument of S_{nmpq}. The result for one element should serve as ample illustration. For a particular sub-term, V_{nmpq}, in the potential, the resulting perturbation in the ascending node, say, is given by:

$$\Delta\Omega_{nmpq} = \mu a_e^n \frac{(\partial F_{nmp}/\partial i) G_{npq} \bar{S}_{nmpq}}{\dot{M} a^{n+3}(1-e^2)^{\frac{1}{2}} \sin i \left[(n-2p)\dot{\omega} + (n-2p+q)\dot{M} + m(\dot{\Omega} - \dot{\theta})\right]}$$

where $\bar{S}_{nmpq}$ is the integral of S_{nmpq} with respect to its argument. The number of $\Delta\Omega_{nmpq}$ necessary to form $\Delta\Omega_{nm}$ is limited by the fact that, among other things, $G_{npq} \sim e^{|q|}$ so that only sub-terms with q near zero need be considered. In practice a subroutine that computes and tests the amplitudes of a sufficiently large range of sub-terms and flags and stores those that are significant within a stated accuracy requirement is used. The amplitudes are normally computed only once (assuming that the orbital elements are known with sufficient accuracy) for the time interval being considered. The perturbations at any time are then obtained by multiplying the amplitudes by the corresponding S_{nmpq} or $\bar{S}_{nmpq}$ computed for that time. This procedure is obviously quite economical in terms of computing time.

Certain interactions between the different perturbations have not been mentioned that slightly complicate matters. Short period perturbations in the semi-major axis will give rise to additional short period perturbations in the mean anomaly that may have to be taken into accourt. In addition, the perturbations in e and i that result from zonal harmonic terms interact with the secular perturbations in ω, Ω, and M due to C_{20}, resulting in additional long period perturbations in ω, Ω, and M that may be important. These effects can, fortunately, be included very easily and this is done in the routines.

At the time of this writing, a completely satisfactory comparison between numerical integrations and analytic theory has not been obtained and no attempt at implementation of the analytic representation in CADNIP has been made. While the two methods show generally good agreement, small differences exist which thus far remain unexplained. Typical results obtained from the two methods for a relatively low orbit with a small eccentricity are illustrated in figures 6 - 1 through 6 - 10. The first 6 figures show the effect of $C_{2,2}$ on the Keplerian elements as determined by the analytic and numeric approaches. The next 3 figures represent the positional displacement implied by the two methods in a rectangular coordinate system whose origin is at the center of the vehicle. Figure 6 - 10 is a magnification of the differences between the two approaches shown in figures 6 - 7 through 6 - 9. The case of very small eccentricity is, of course, quite important in applications of CADNIP. The differences decrease for larger eccentricity but do not entirely disappear until eccentricities on the order of 0.1 are reached. These computations were made with C_{20} included in the reference integration used in determining the perturbations under numerical integration. An interesting result is that, when C_{20} is not included, excellent agreement between the two methods seems to occur. This is illustrated in figures 6 - 11 through 6 - 20. It would appear, then, that the observed differences might be due to interactions between the short-periodic perturbations due to C_{20} and the higher-order terms that are not included in the analytic theory. The short-periodic perturbations in ω, M and (relatively) e due to C_{20} become quite large for small eccentricity. This hypothesis was tested in considerable detail. Such interaction terms can be computed from:

$$\Delta_2 s_i = \sum_j \frac{\partial s_i}{\partial s_j} \Delta_1 s_j$$

where s_i is any one of the Keplerian elements being considered with respect to a particular sub-term and $\Delta_1 s_j$ is the interacting perturbation in any one of the elements. The $\partial s_i / \partial s_j$'s can be obtained by differentiation of the Lagrangian equations of motion after substitution of the various derivatives of the potential with respect to the elements, and the $\Delta_1 s_j$ can be obtained via the first-order theory outlined above. This is quite simple in theory but extremely laborious in practice, both from an algebraic and a logical point of view. Nonetheless, the routines were modified to include the interaction terms, in both directions, between the short-periodic perturbations due to C_{20} and any particular higher-order term.

The result of this work was that the interaction terms showed rather small amplitudes (although they were beginning to be significant in terms of the most accurate observations) even for low eccentricity, and were not as large as the observed differences. Moreover, they were strictly periodic as they clearly should be. The observed differences, on the other hand, are somewhat irregular and tend to grow with time. This gives some reason to suspect the numerical integration. Careful checking and comparison with an essentially independent formulation have, however, failed to reveal any fault in either the theory or coding of the routines involved in the integration.

It is felt that the possibility of analytic representation of the high-order terms in the geopotential should be explored further. It may well be that what we have observed is the beginning of failure of the theory in the case of near zero eccentricity. Such failure must eventually occur when the Keplerian elements are used. If this is the case, the problem, which would be of a purely mathematical nature, could be solved by a proper transformation of the orbital elements and development in terms of the Delaunay elements. This would, clearly, involve a considerable amount of analysis and programming. Of course, if this were determined to be the cause of the difficulty, limits might be established and the existing routines incorporated in CADNIP for use with larger eccentricities. Perhaps consideration should be given to both of these approaches in future work.

Figure 6 - 1

M	OMEGA	C OMEGA	I	E	A
0.000	0.000	0.000	45.0000	.00500	7136.7028

PERIOD (MINUTES) = 100.00

RUN FOR C(2.2)

(✱) NUMERIC
(+) ANALYTIC
(⊙) DIFFERENCE

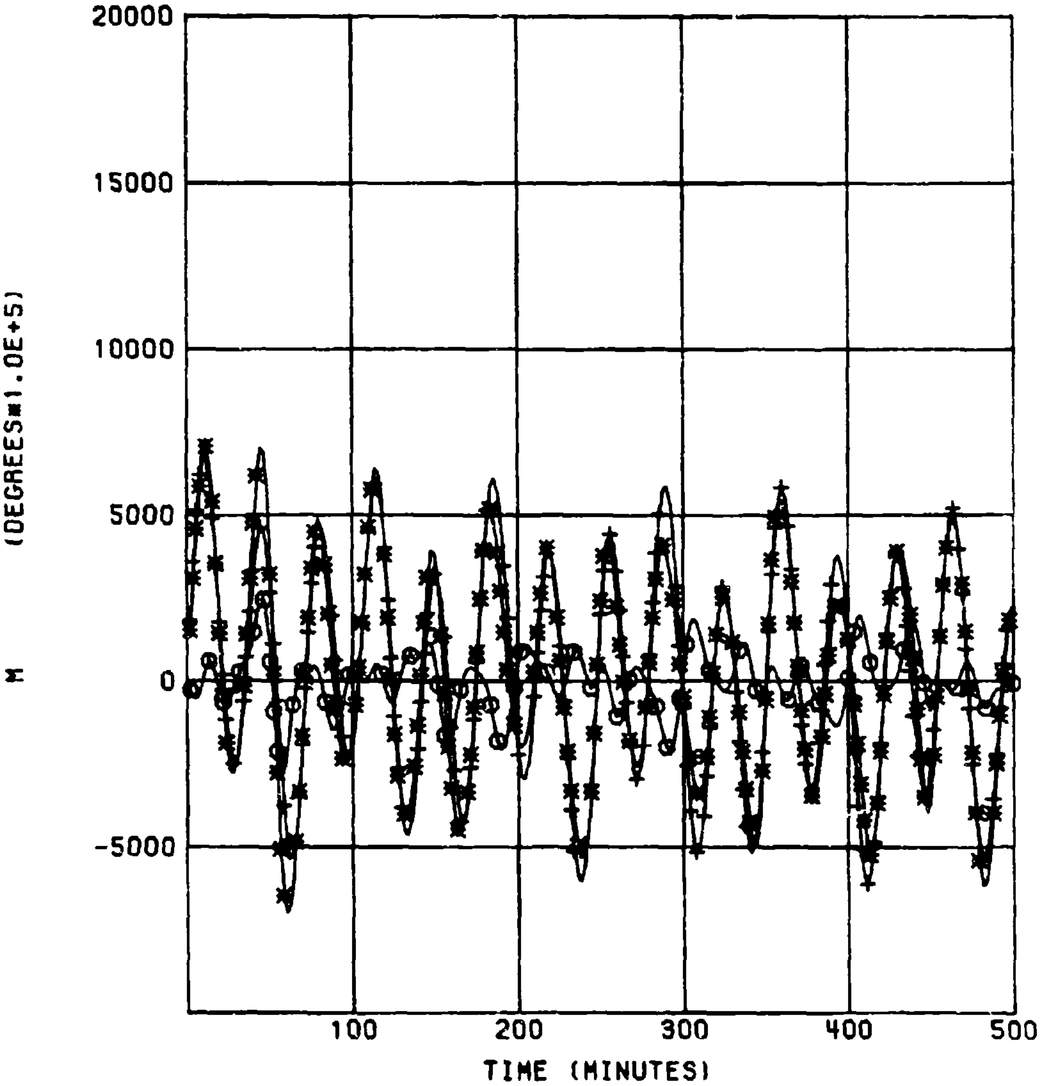


Figure 6 - 2

M	OMEGA	C OMEGA	I	E	A
0.000	0.000	0.000	45.0000	.00500	7136.7028

PERIOD (MINUTES) = 100.00

RUN FOR C(2,2)

(✱) NUMERIC
(+) ANALYTIC
(⊙) DIFFERENCE

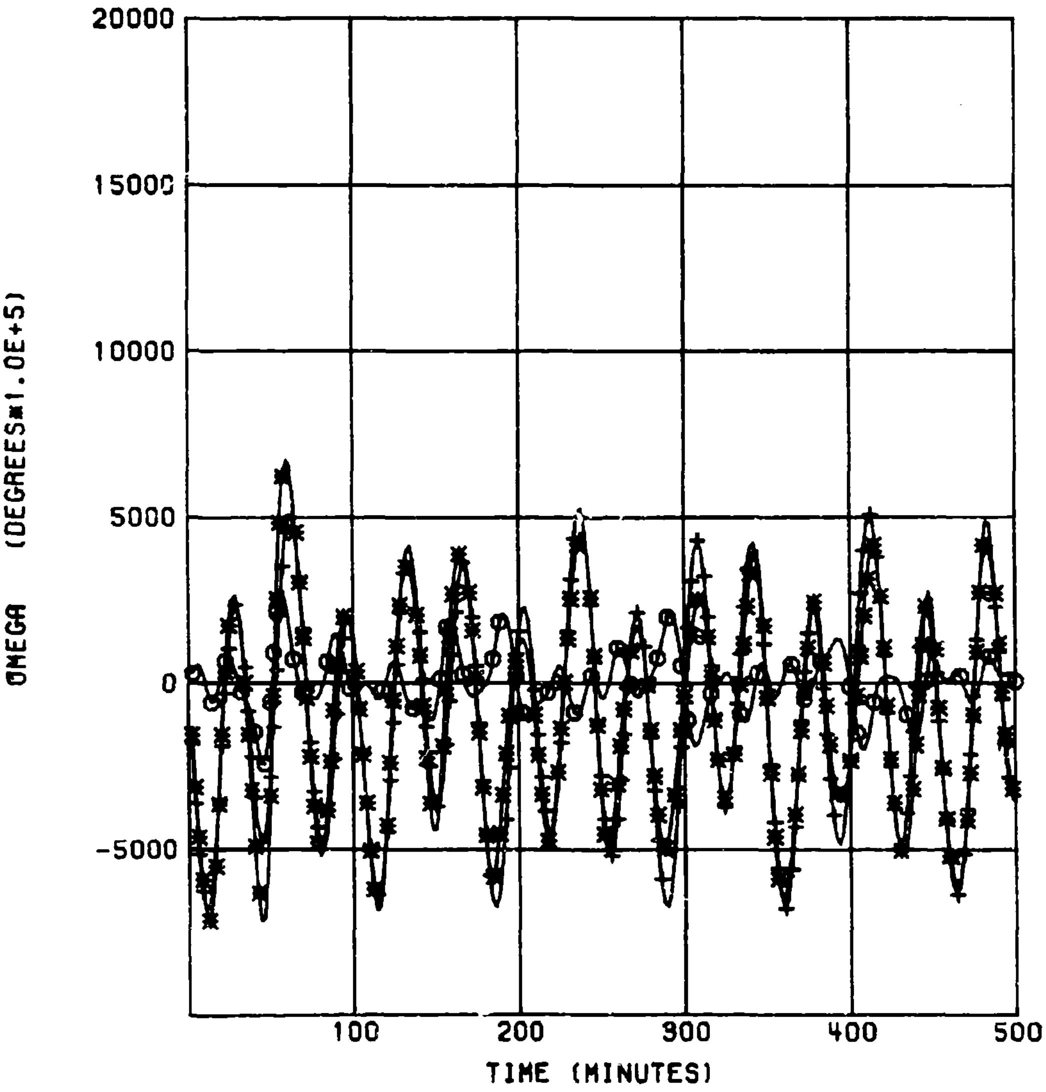


Figure 6 - 3

M	OMEGA	C OMEGA	I	E	A
0.000	0.000	0.000	45.0000	.00500	7136.7028

PERIOD (MINUTES) = 100.00

RUN FOR C(2.2)

(✱) NUMERIC
(+) ANALYTIC
(O) DIFFERENCE

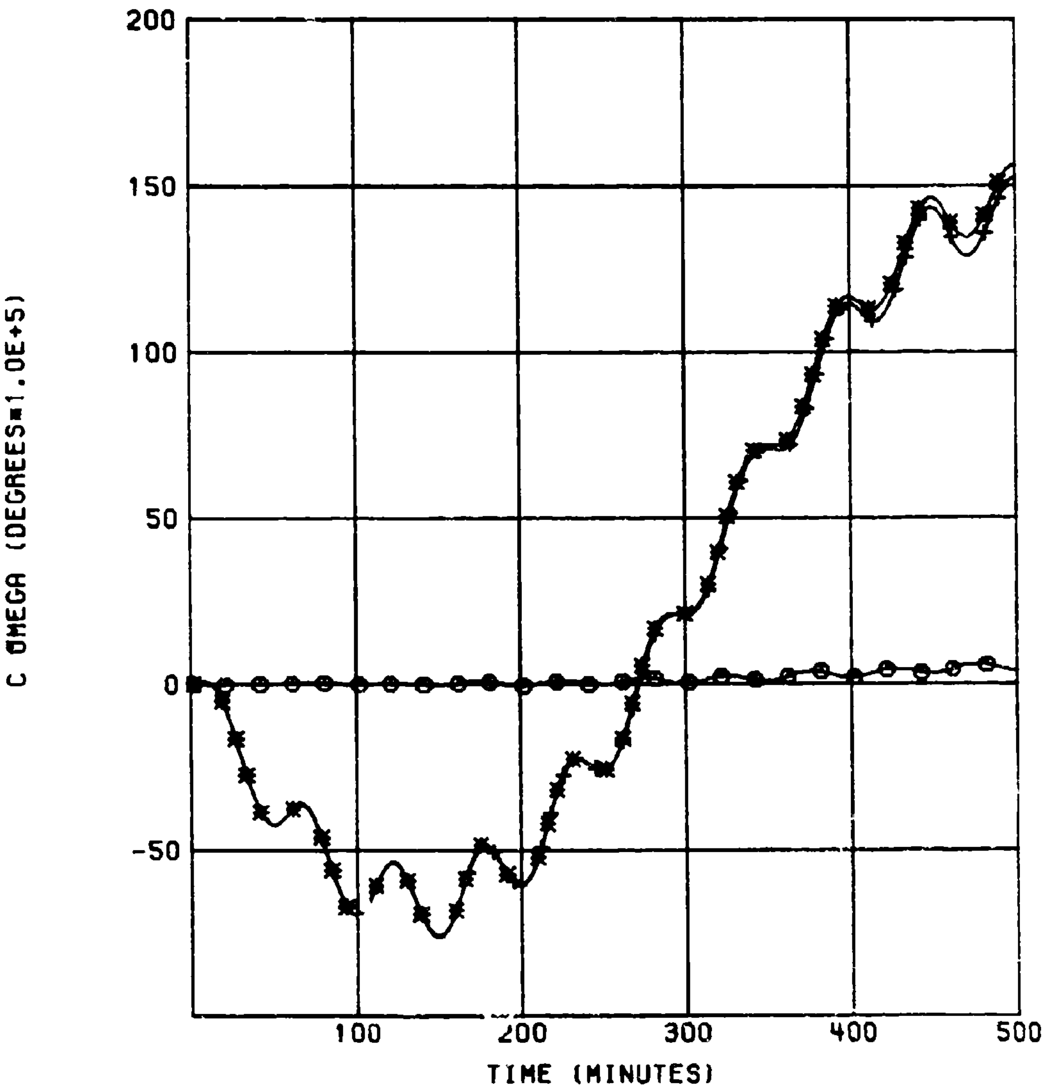


Figure 6 - 4

M	OMEGA	C OMEGA	I	E	A
0.000	0.000	0.000	45.0000	.00500	7136.7028

PERIOD (MINUTES) = 100.00

RUN FOR C(2,2)

(✱) NUMERIC
(+) ANALYTIC
(O) DIFFERENCE

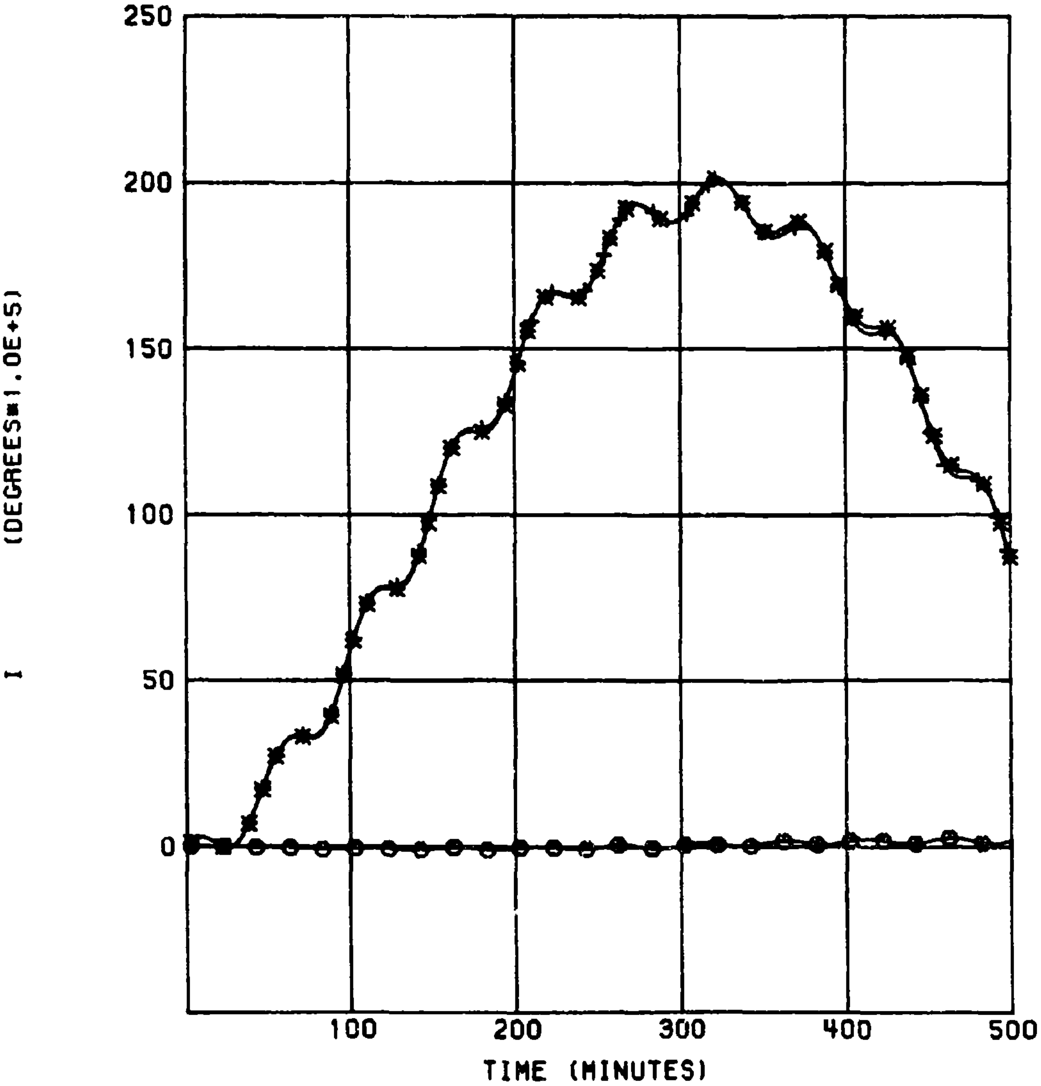


Figure 6 - 5

M	OMEGA	C OMEGA	I	E	A
0.000	0.000	0.000	45.0000	.00500	7136.7028

PERIOD (MINUTES) = 100.00

RUN FOR C(2,2)

(✱) NUMERIC
(+) ANALYTIC
(O) DIFFERENCE

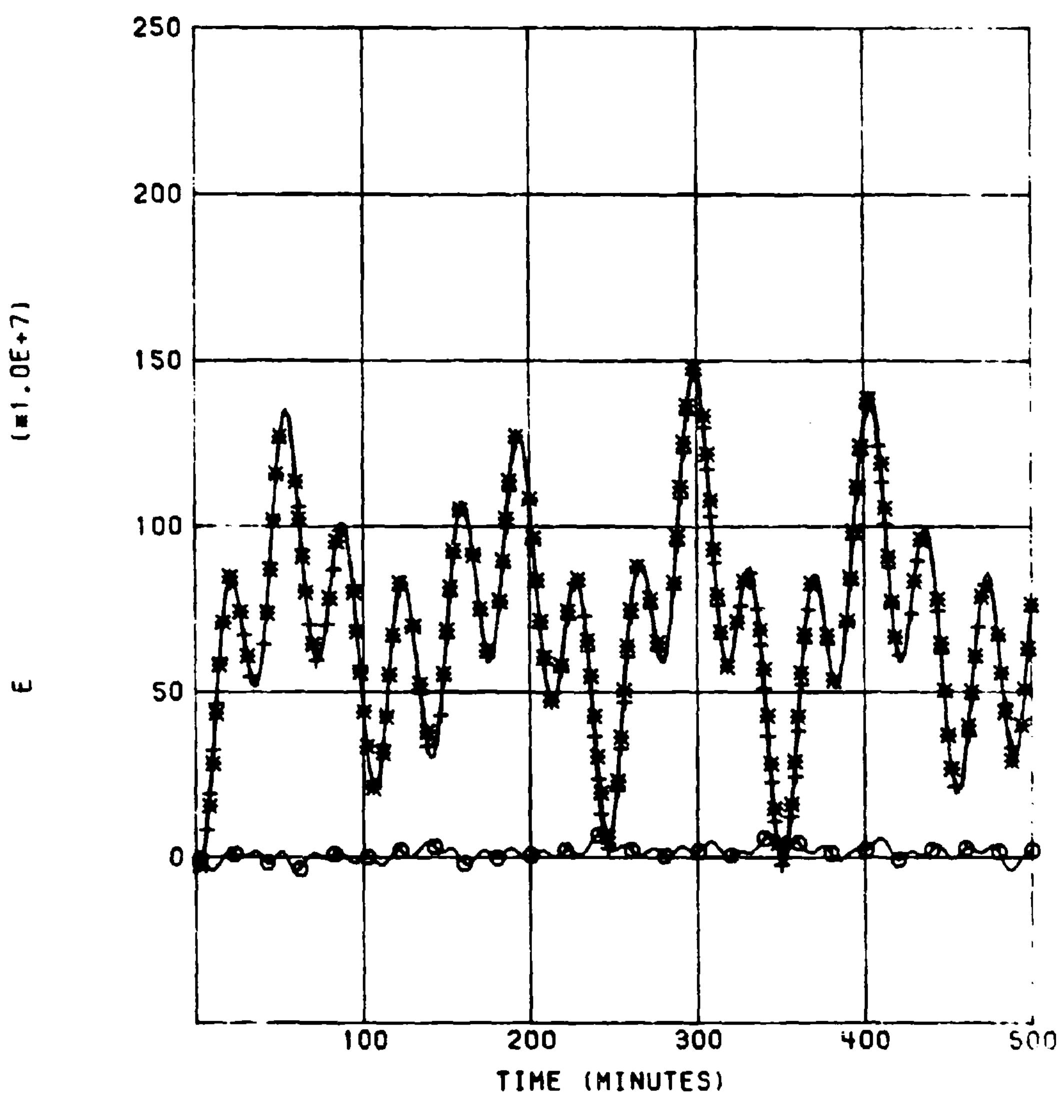


Figure 6 - 6

M	OMEGA	C OMEGA	I	E	A
0.000	0.000	0.000	45.0000	.00500	7136.7028

PERIOD (MINUTES) = 100.00

RUN FOR C(2.2)

(✱) NUMERIC
(+) ANALYTIC
(O) DIFFERENCE

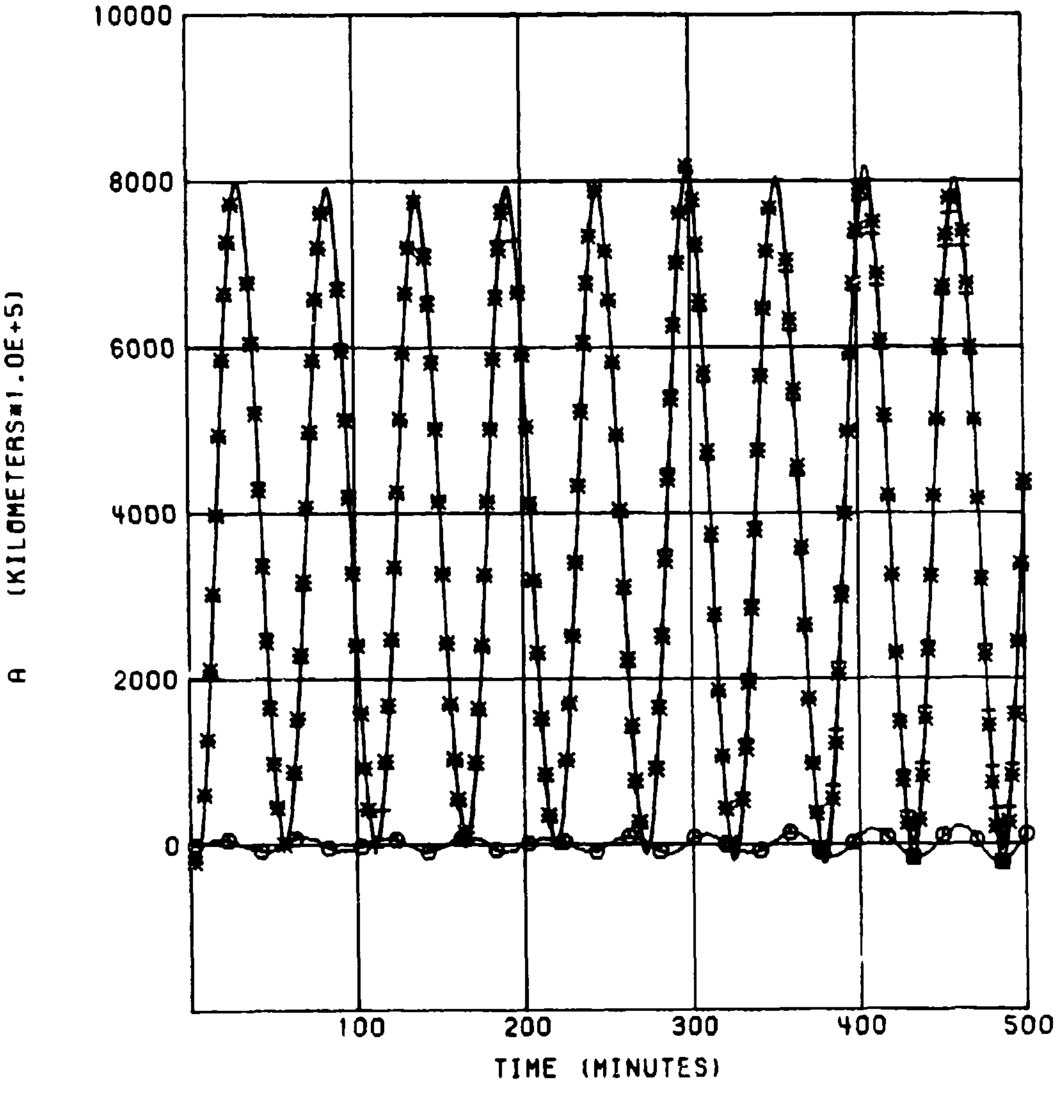


Figure 6 - 7

M	OMEGA	C OMEGA	I	E	A
0.000	0.000	0.000	45.0000	.00500	7136.7028

PERIOD (MINUTES) = 100.00

RUN FOR C(2,2)

(✱) NUMERIC
(+) ANALYTIC
(O) DIFFERENCE

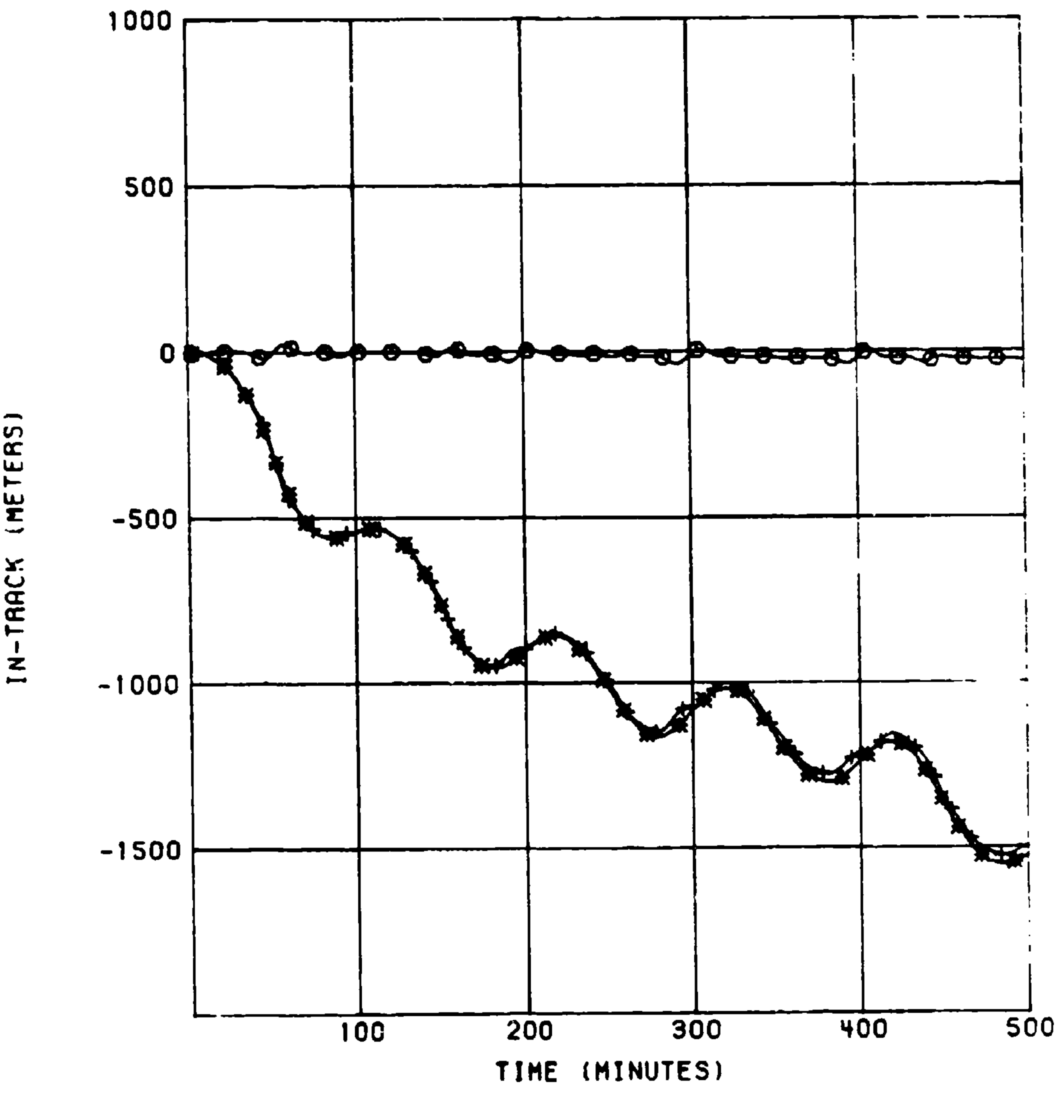


Figure 6 - 8

M	OMEGA	C OMEGA	I	E	A
0.000	0.000	0.000	45.0000	.00500	7136.7028

PERIOD (MINUTES) = 100.00

RUN FOR C(2.2)

(✱) NUMERIC
(+) ANALYTIC
(Ⓞ) DIFFERENCE

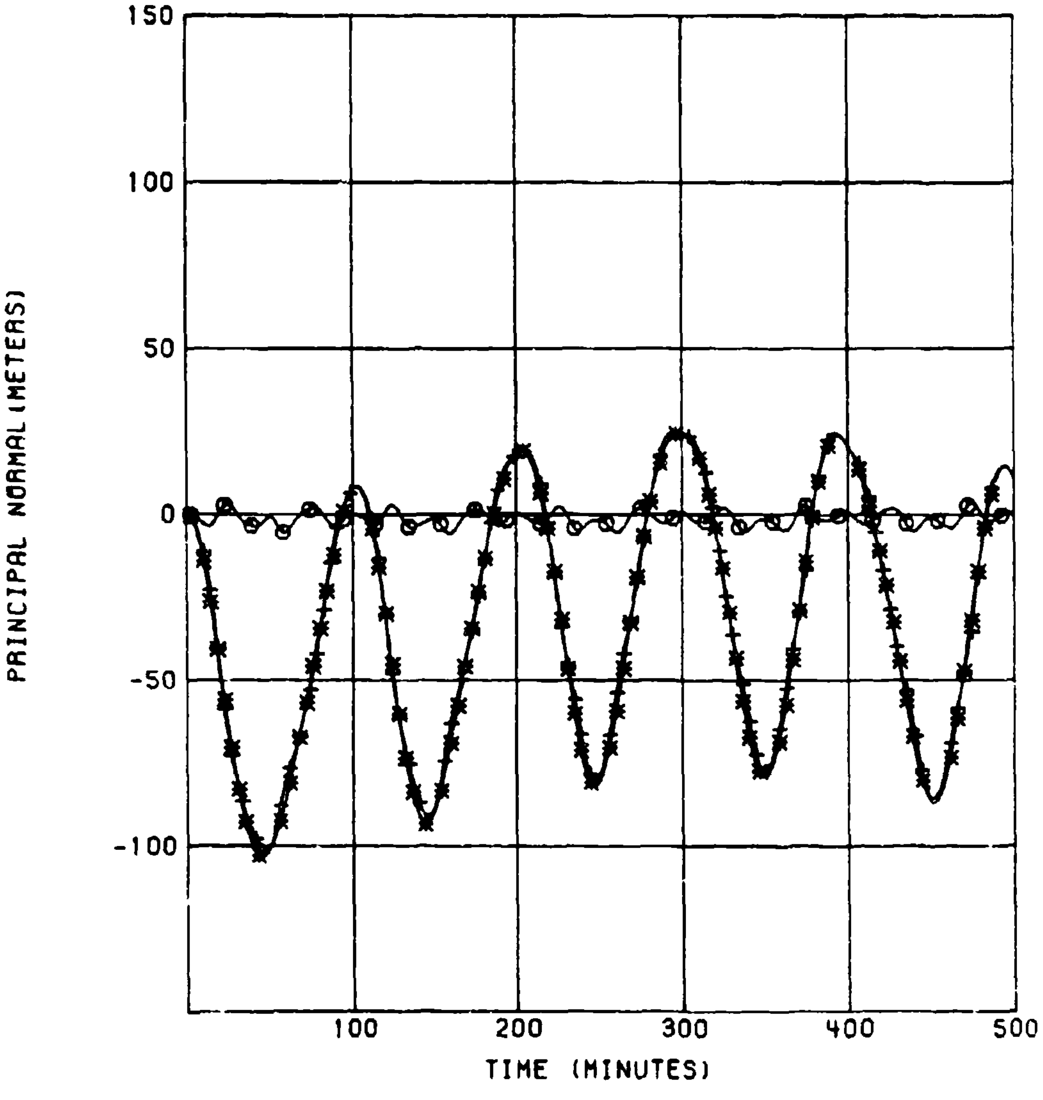


Figure 6 - 9

M	OMEGA	C OMEGA	I	E	A
0.000	0.000	0.000	45.0000	.00500	7136.7028

PERIOD (MINUTES) = 100.00

RUN FOR C(2,2)

(✱) NUMERIC
(+) ANALYTIC
(O) DIFFERENCE

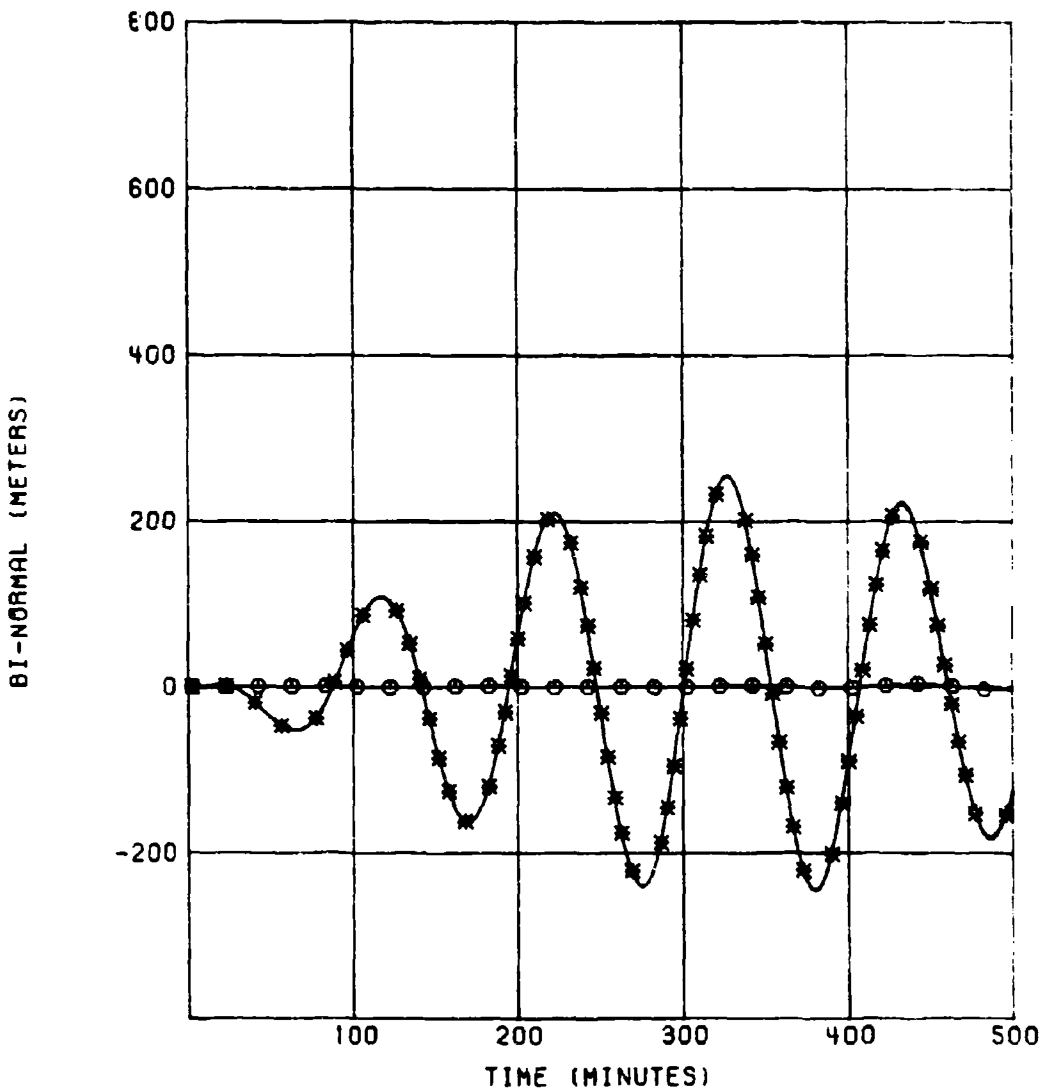


Figure 6 - 10

M	OMEGA	C OMEGA	I	E	A
0.000	0.000	0.000	45.0000	.00500	7136.7028

PERIOD (MINUTES) = 100.00

RUN FOR C(2,2)

(✱) IN-TRACK
(+) PRINCIPAL NORMAL CROSS-TRACK
(⊙) BI-NORMAL CROSS-TRACK

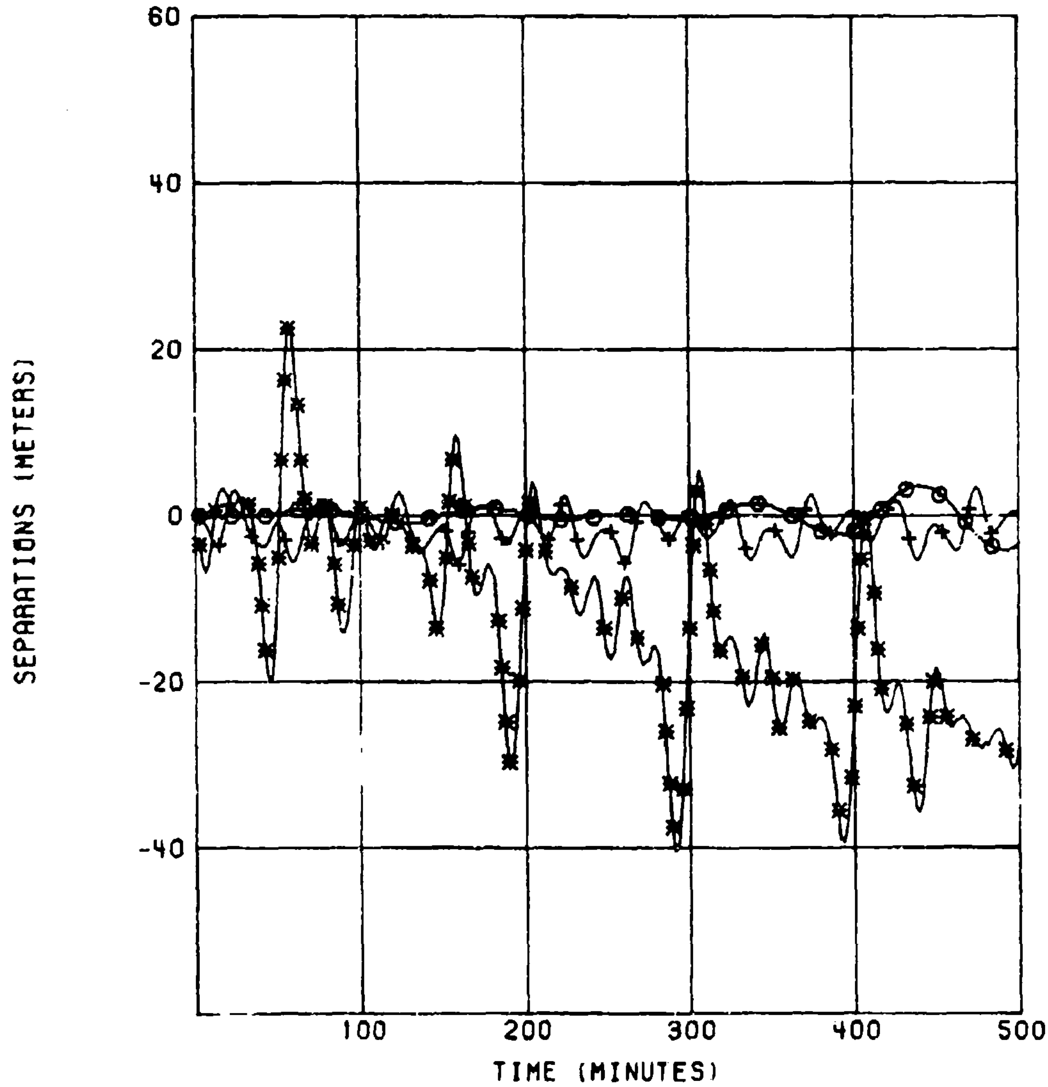


Figure 6 - 11

M	OMEGA	C OMEGA	I	E	A
0.000	0.000	0.000	45.0000	.00500	7136.7028

PERIOD (MINUTES) = 100.00

RUN FOR C(2,2)

(*) NUMERIC
(+) ANALYTIC
(O) DIFFERENCE

J(2) ELIMINATED

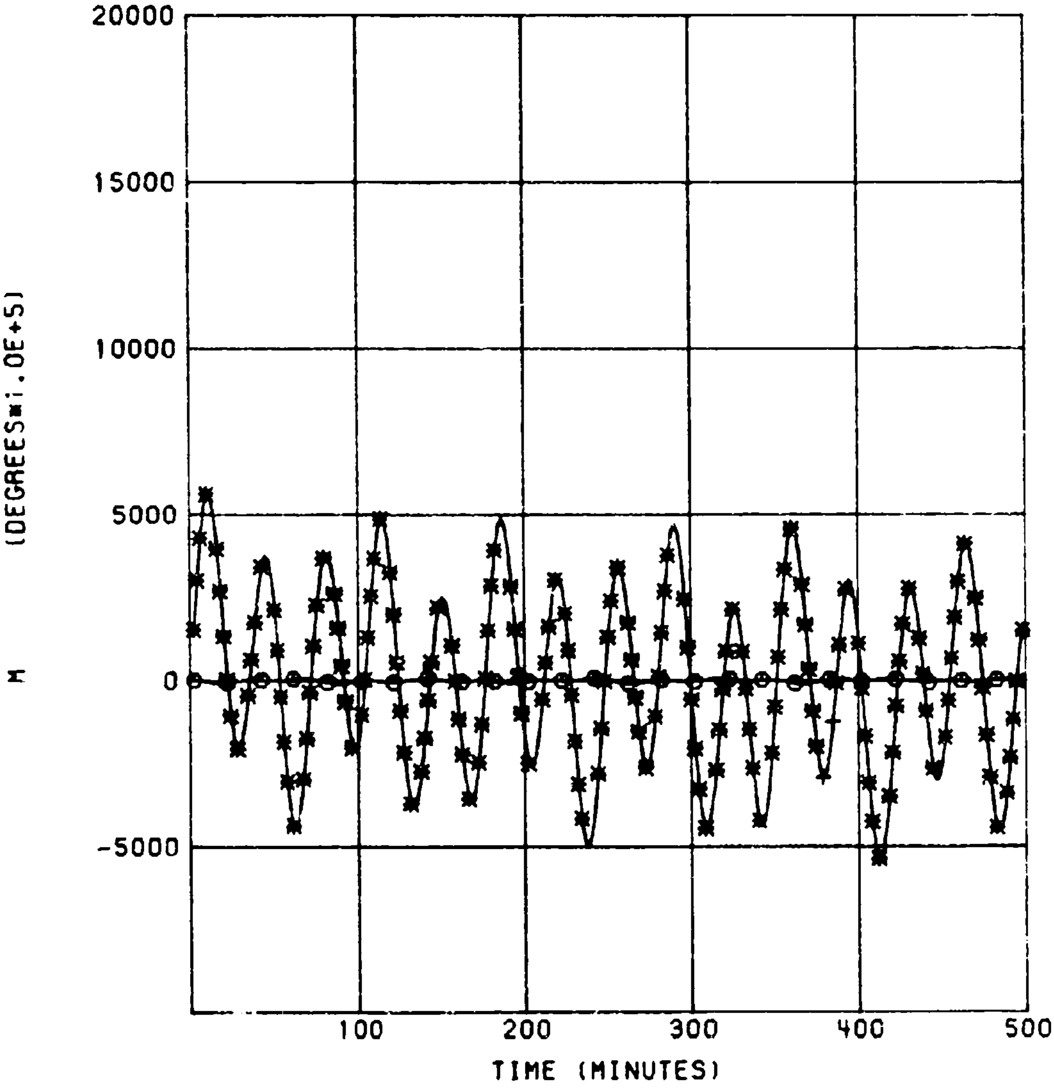


Figure 6 - 12

M	OMEGA	C OMEGA	I	E	A
0.000	0.000	0.000	45.0000	.00500	7136.7028

PERIOD (MINUTES) = 100.00

RUN FOR C(2,2)

(*) NUMERIC
(+) ANALYTIC
(O) DIFFERENCE

J(2) ELIMINATED

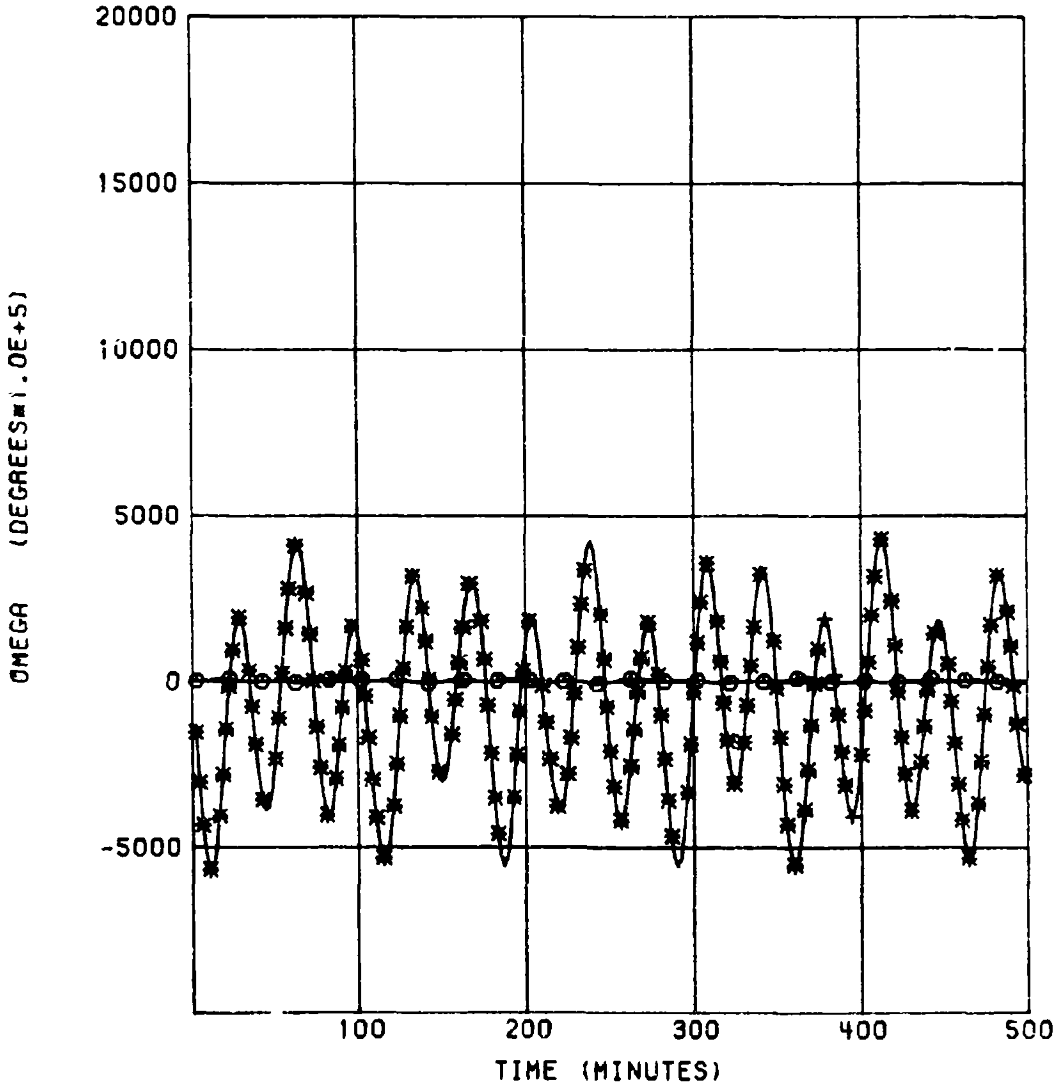


Figure 6 - 13

M	OMEGA	C OMEGA	I	E	A
0.00J	J.000	0.000	45.0000	.00500	7136.7028

PERIOD (MINUTES) = 100.00

PUN FOR C(2,2)

(*) NUMERIC
(+) ANALYTIC
(O) DIFFERENCE

J(2) ELIMINATED

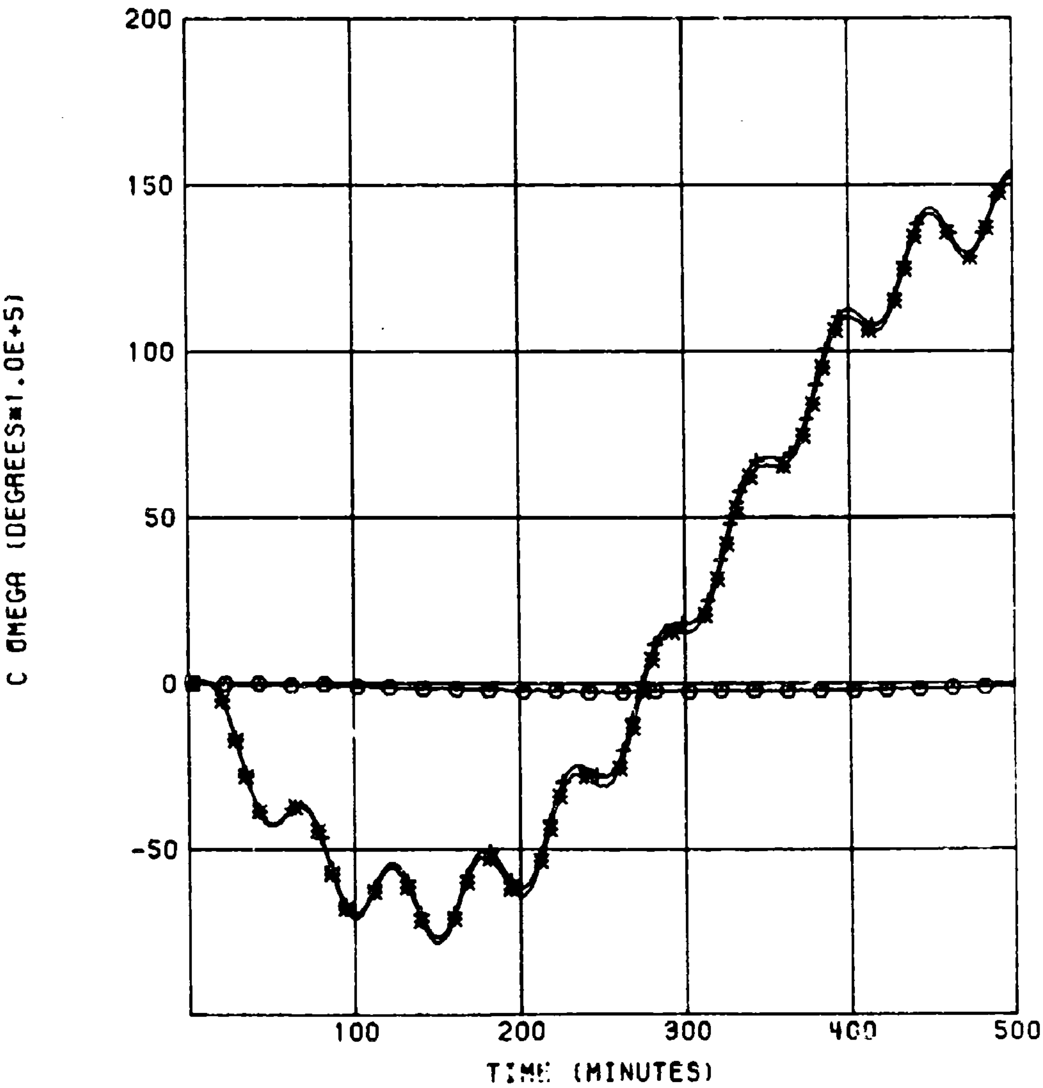


Figure 6 - 14

M	OMEGA	C OMEGA	I	E	A
0.000	0.000	0.000	45.0000	.00500	7136.7028

PERIOD (MINUTES) = 100.00

RUN FOR C(2.2)

(*) NUMERIC
(+) ANALYTIC
(O) DIFFERENCE

J(2) ELIMINATED

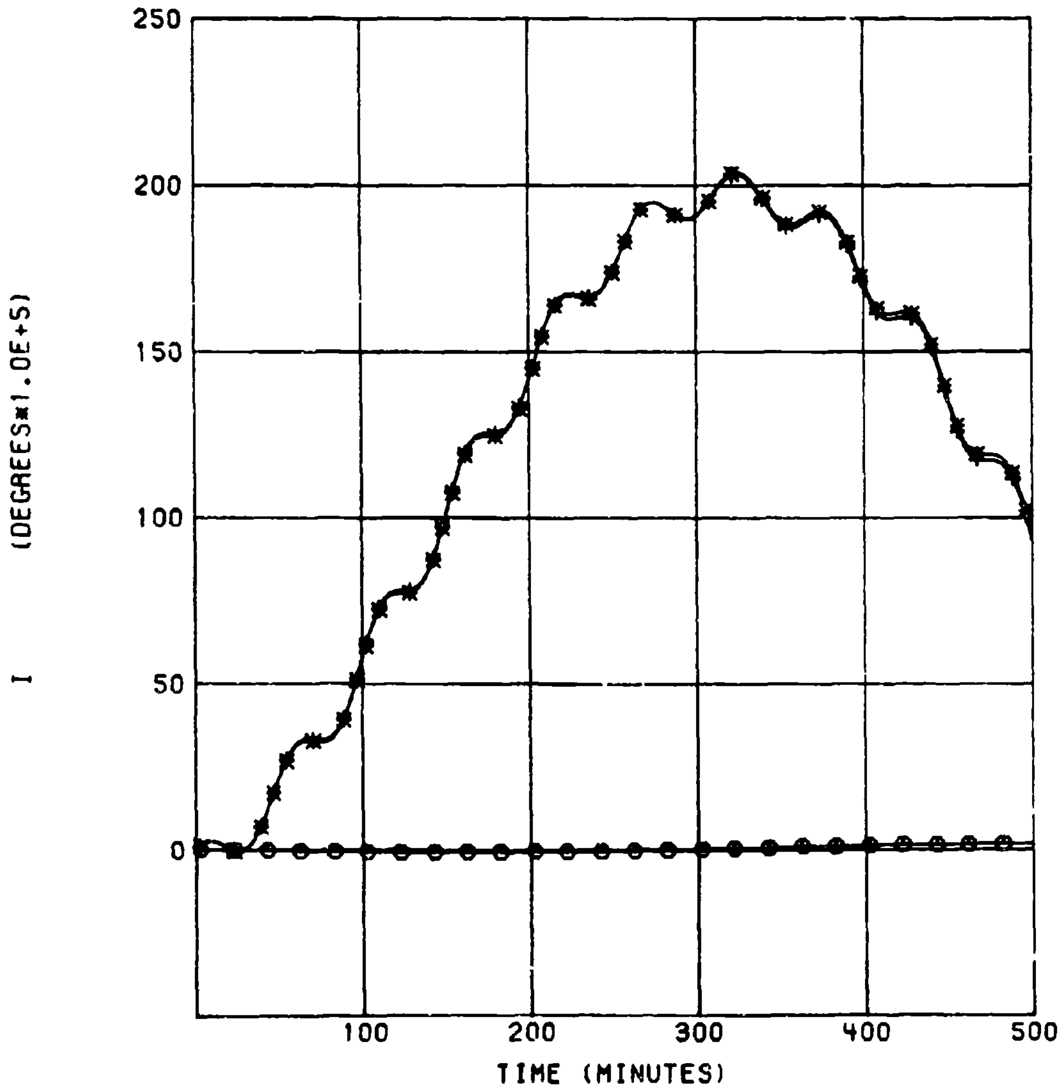


Figure 6 - 15

M	OMEGA	C OMEGA	I	E	A
0.000	0.000	0.000	45.0000	.00500	7136.7028

PERIOD (MINUTES) = 100.00

RUN FOR C(2,2)

(✱) NUMERIC
(+) ANALYTIC
(⊙) DIFFERENCE

J(2) ELIMINATED

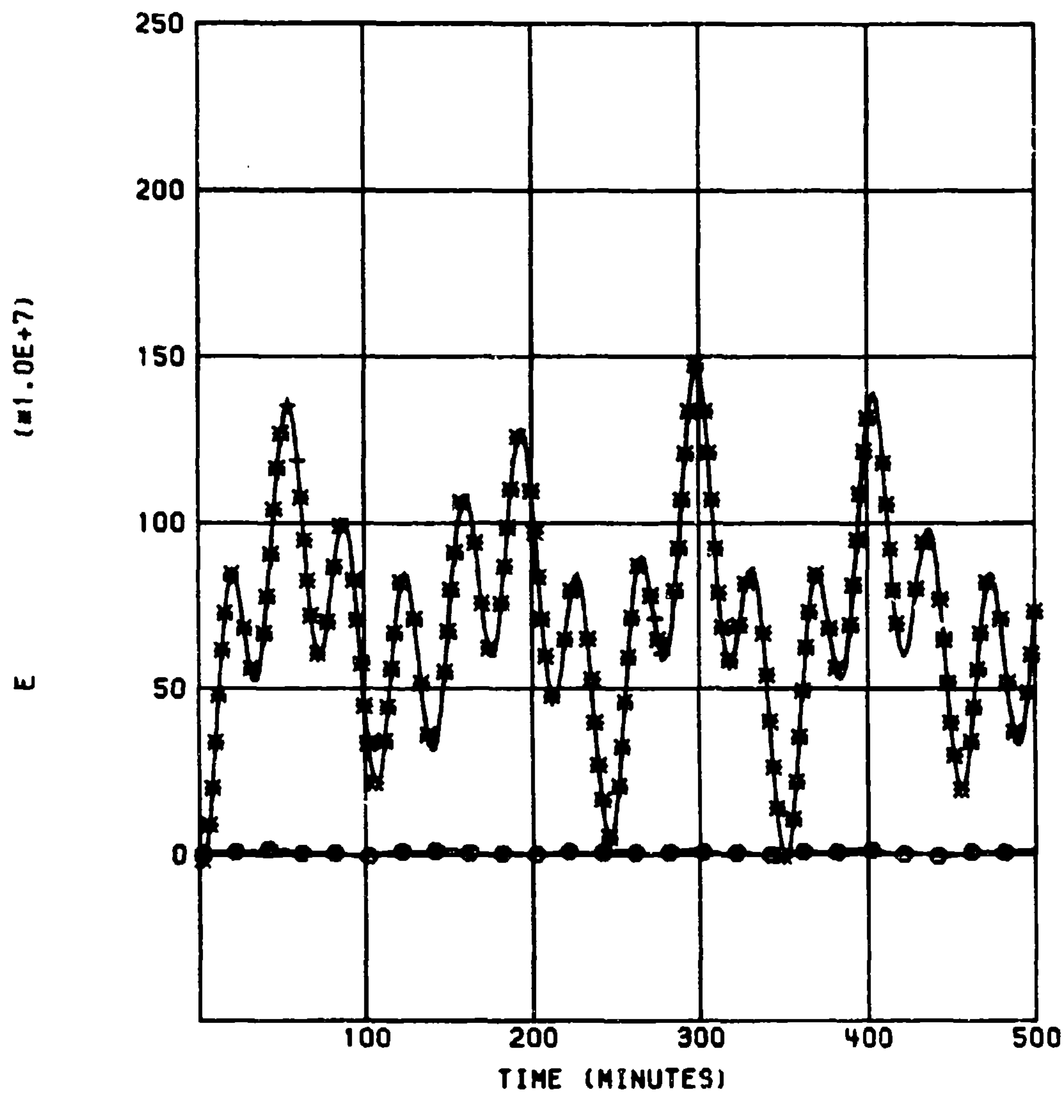


Figure 6 - 16

M	OMEGA	C OMEGA	I	E	A
0.000	0.000	0.000	45.0000	.00500	7136.7028

PERIOD (MINUTES) = 100.00

RUN FOR C(2,2)

(✱) NUMERIC
(+) ANALYTIC
(O) DIFFERENCE

J(2) ELIMINATED

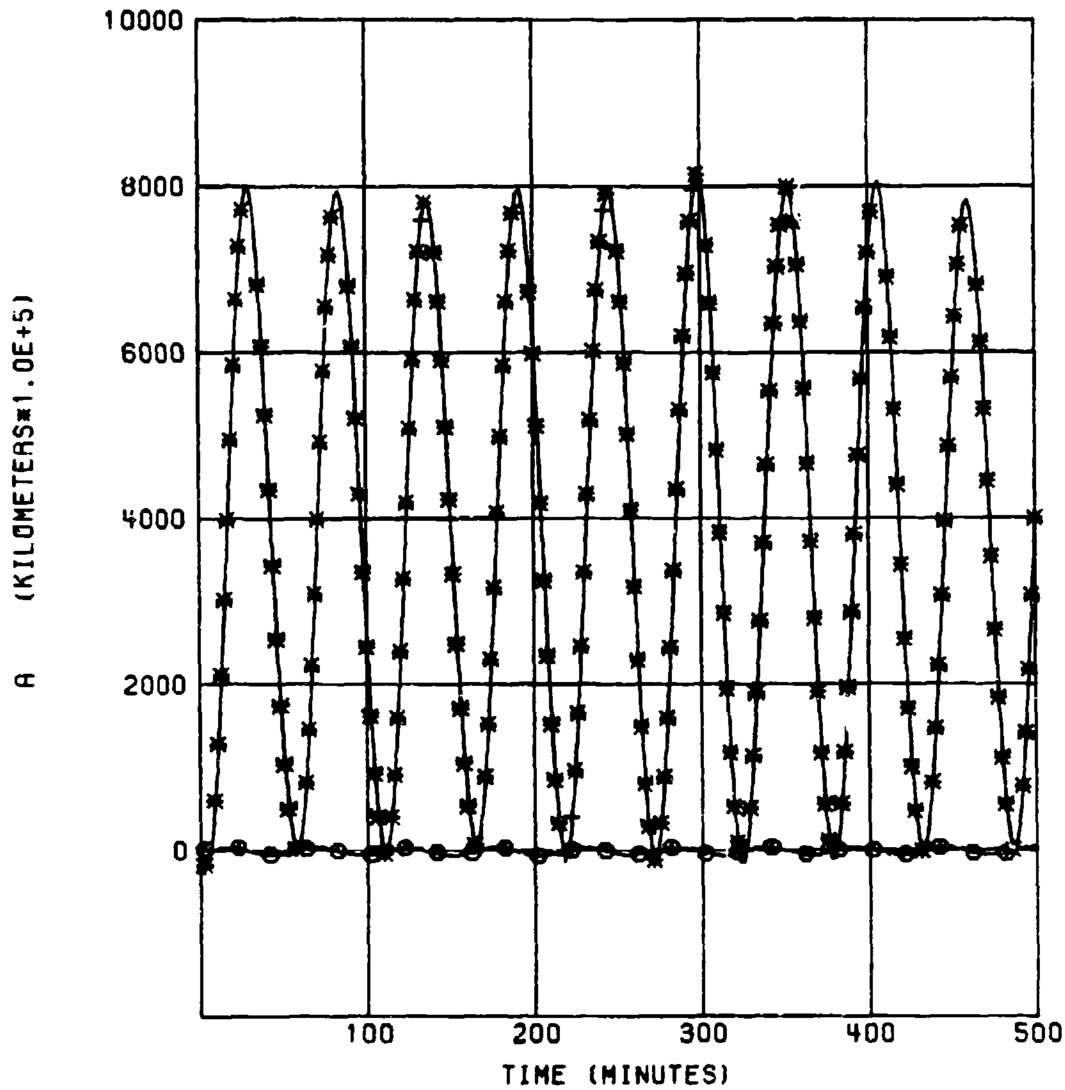


Figure 6 - 17

M	OMEGA	C OMEGA	I	E	A
0.000	0.000	0.000	45.0000	.00500	7136.7028

PERIOD (MINUTES) = 100.00

RUN FOR C(2,2)

(✱) NUMERIC
(+) ANALYTIC
(O) DIFFERENCE

J(2) ELIMINATED

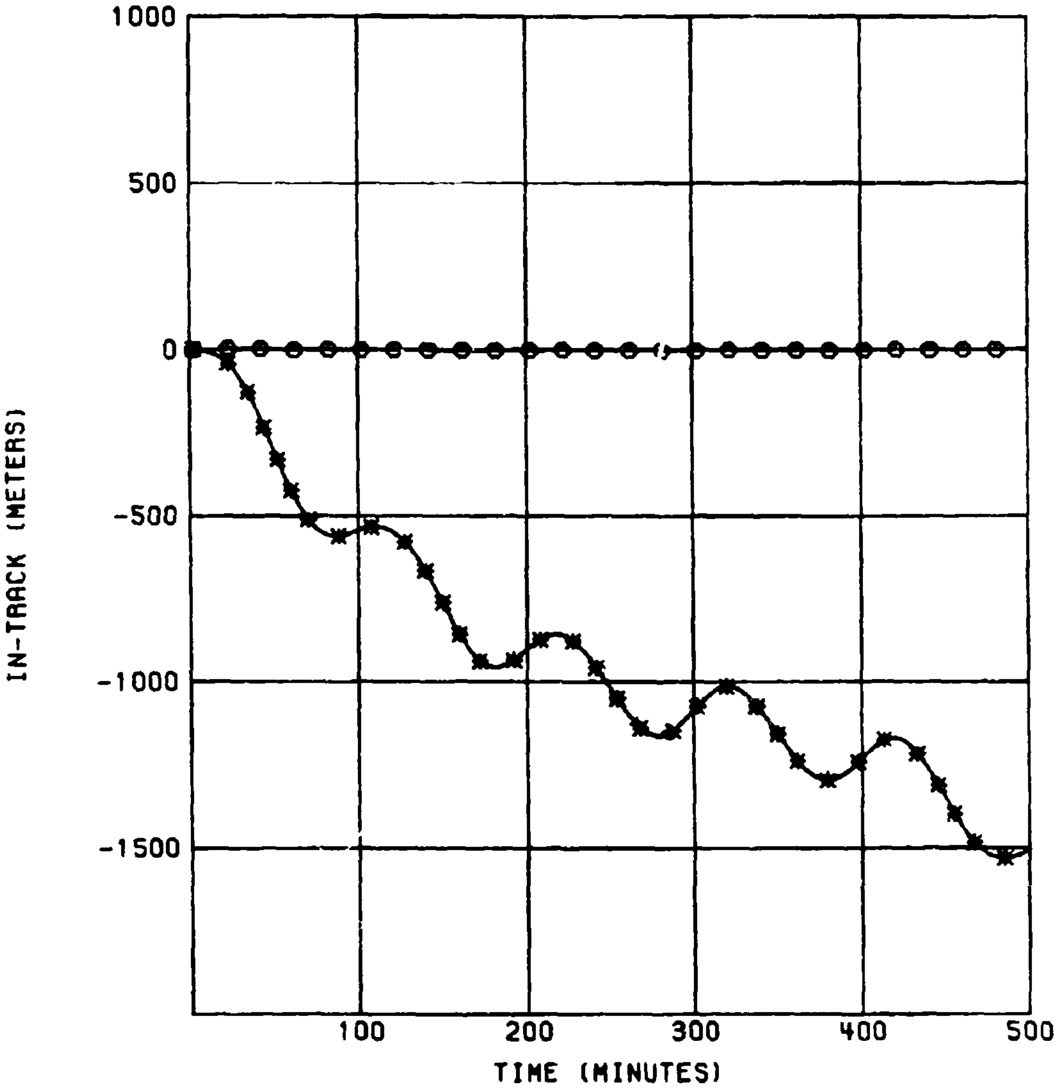


Figure 6 - 18

M	OMEGA	C OMEGA	I	E	A
0.000	0.000	0.000	45.0000	.00500	7136.7028

PERIOD (MINUTES) = 100.00

RUN FOR C(2,2)

(✱) NUMERIC
(+) ANALYTIC
(⊙) DIFFERENCE

J(2) ELIMINATED

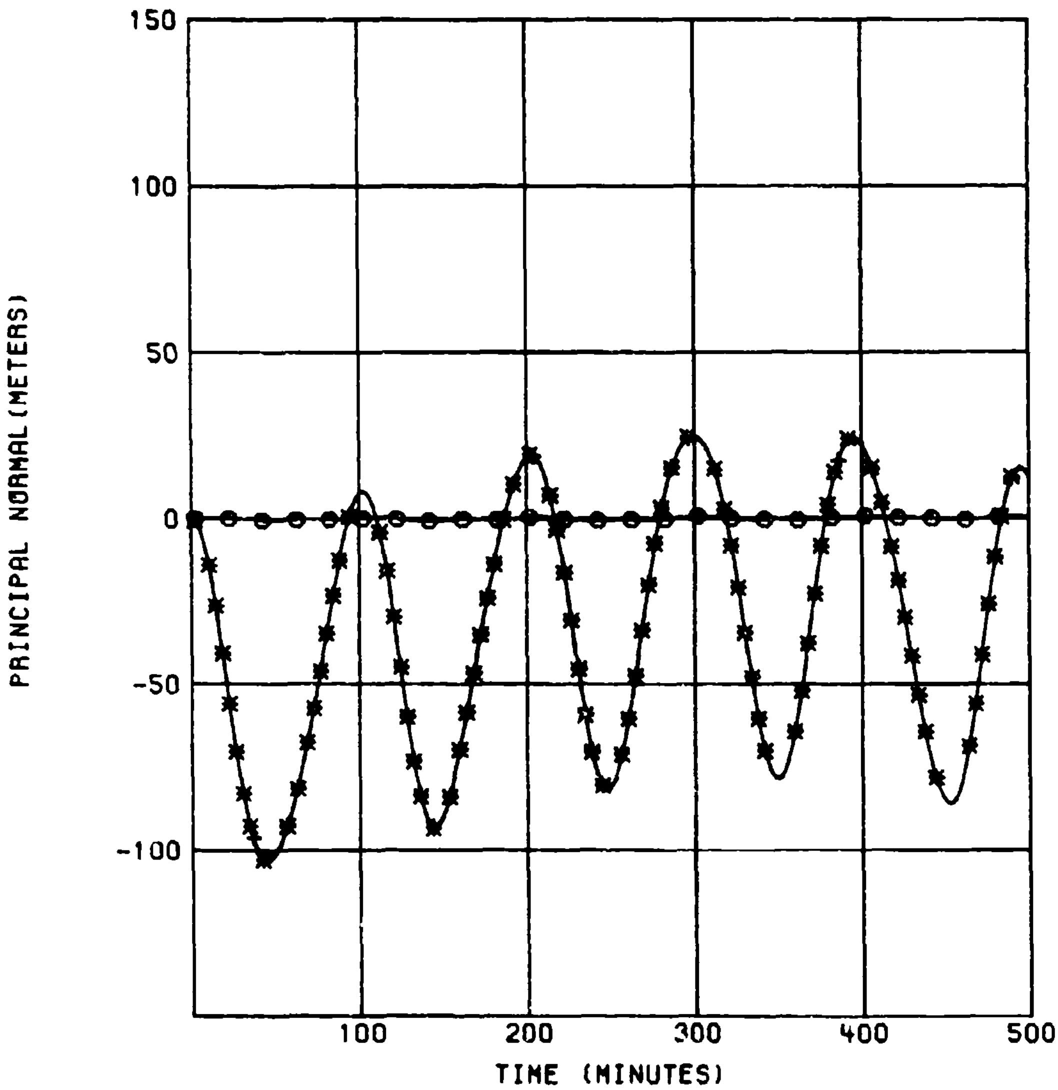


Figure 6 - 19

M	OMEGA	C OMEGA	I	E	A
0.000	0.000	0.000	45.0000	.00500	7136.7028

PERIOD (MINUTES) = 100.00

RUN FOR C(2.2)

(✱) NUMERIC
(+) ANALYTIC
(⊕) DIFFERENCE

J(2) ELIMINATED

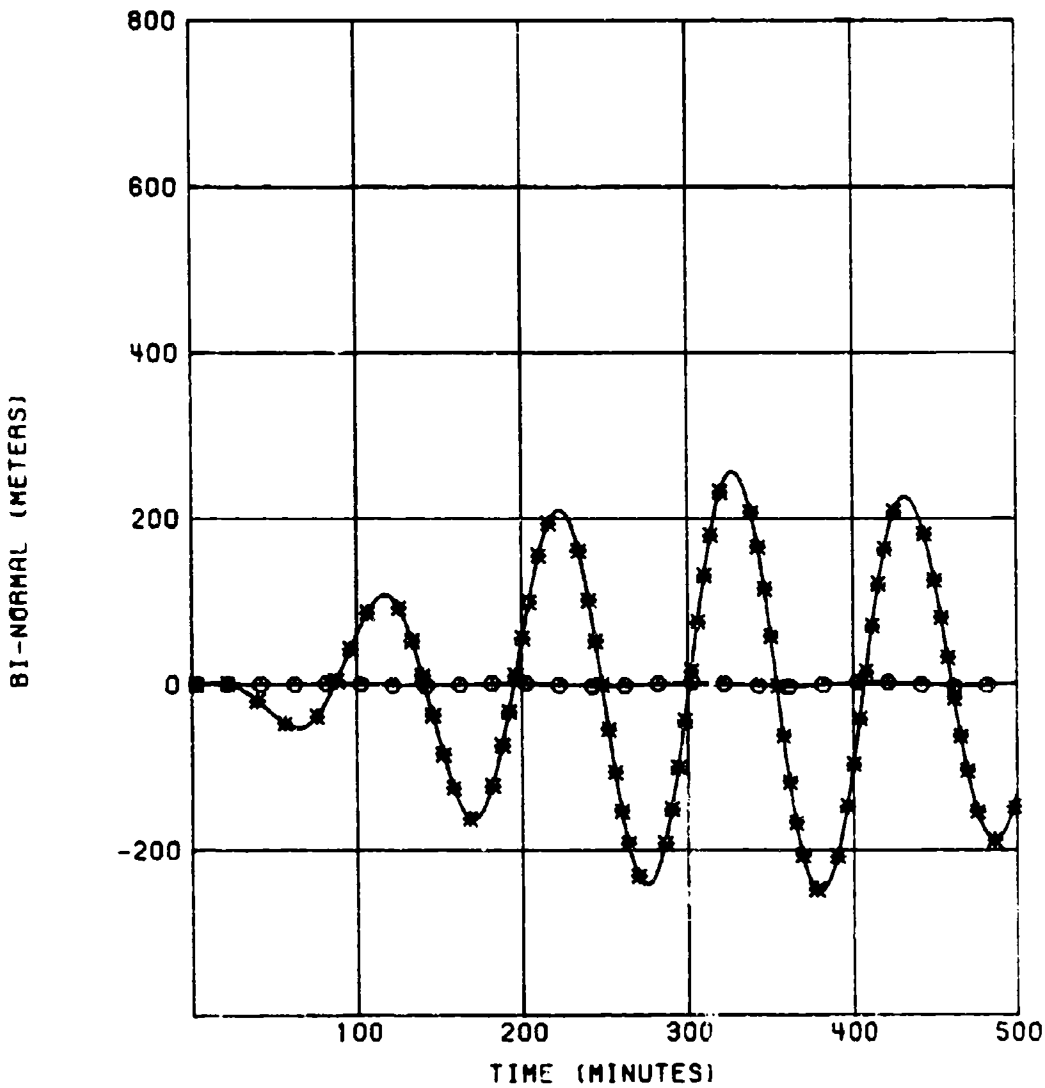


Figure 6 - 20

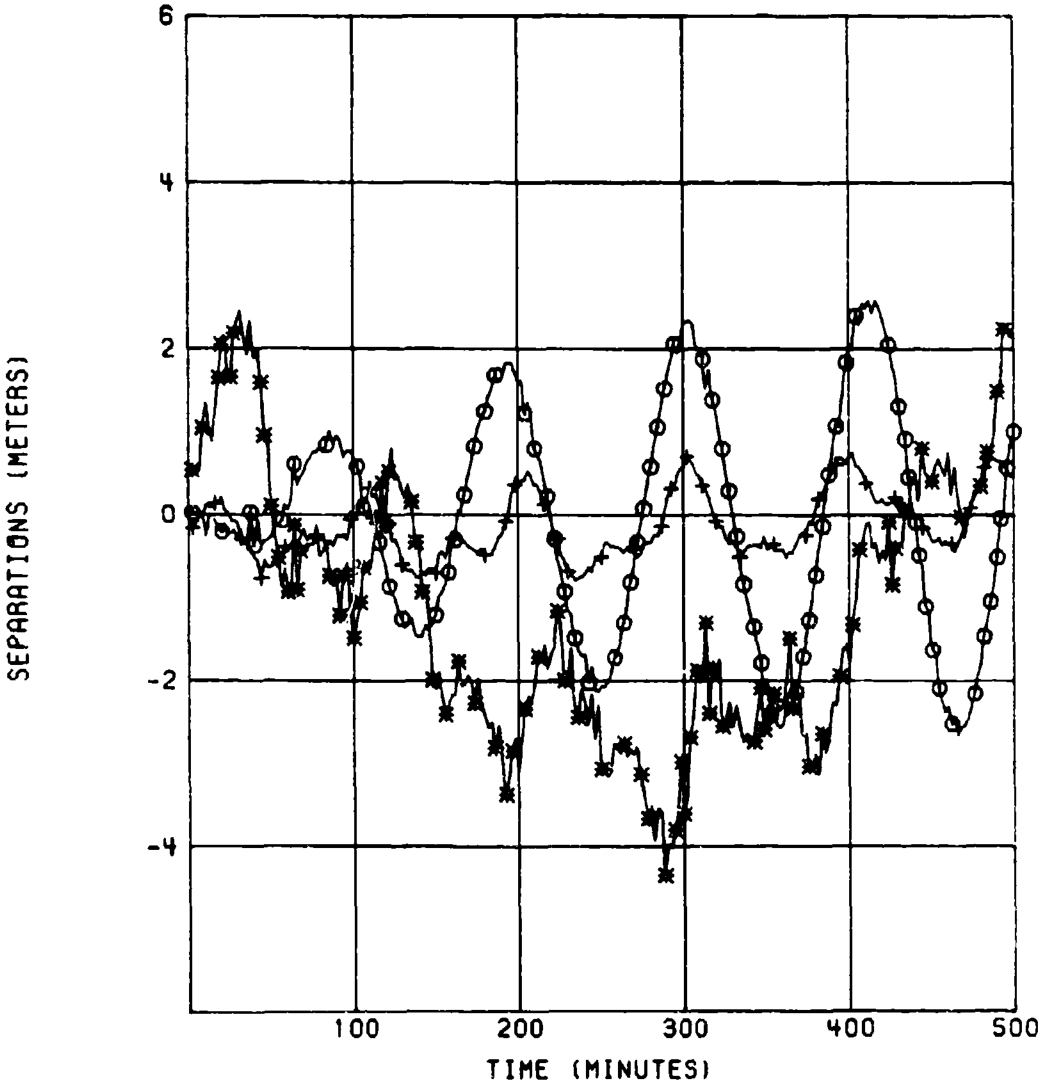
M	OMEGA	C OMEGA	I	E	A
0.000	0.000	0.000	45.0000	.00500	7136.7028

PERIOD (MINUTES) = 100.00

RUN FOR C(2,2)

(*) IN-TRACK
(+) PRINCIPAL NORMAL CROSS-TRACK
(O) BI-NORMAL CROSS-TRACK

J(2) ELIMINATED



Section 7

Recommendations

The program developed under this contract has proven to be a valuable tool in atmospheric density research. The improvements and refinements incorporated in CADNIP have resulted in a significant reduction in computer running time while yielding more accurate and reliable results. One area of the research effort has been disappointing however. The inability to successfully incorporate in CADNIP the analytic formulation for the geopotential has been a source of anguish and frustration. Considering the time and effort already devoted to this, and considering the savings in computer time that would be realized if this were successful, it would seem that additional effort in this direction as outlined in section 6 is warranted.

An additional area of development which is recommended is the inclusion of more recent and sophisticated atmospheric models in the program. This would be relatively easy to accomplish since the mechanism or logic for the insertion of new models was essentially incorporated in CADNIP when the drag model described in section 5.7 was introduced. Alternatively, the CADNIP program may be used as a test vehicle for atmospheric models. Given a set of observations, the program would be run using a number of different models. The model which yields an ephemeris closest to known positions would be considered most representative of the atmosphere.

In the present version of the CADNIP program, the processing of a set of observations is terminated by the completion of the differential correction procedure. It is possible however under certain circumstances to deduce additional density information from the observations. A plot of the mean anomy residuals (which are expected to appear as random noise) may reveal trends which could yield additional density data. Thus it is suggested that a post-processor be included in CADNIP which will examine the final residuals in an attempt to isolate any previously ignored data such as time dependent density information.

Appendix A

USER'S MANUAL

This section of the report is concerned with the actual use of the program and includes descriptions of all input data as well as operating instructions.

The program requires four types of data for every run: satellite observations; station coordinates and related data; a model atmosphere; and control cards which specify the parameters of a run.

A.1 Observation Preparation

The present version of the program is capable of processing observational data prepared in one of three formats; Smithsonian format; AFCRL format; or SPADATS format.

A.1.1 Smithsonian Format

Each observation is punched onto a card which contains the following information. (This description will be limited to the portion of the card interpreted by the program. A complete description may be found in reference 9.)

Field	Columns	Contents-Description
1	14-17	Station number.
2	18-23	Year, month and day of the observation with two columns for each.
3	24-25	Hour of observation.
4	26-27	Minute of observation.
5	28-33	Seconds of observation. The decimal point is assumed to be between columns 29 and 30.
6	34-36	Degrees portion of azimuth or hours portion of right ascension.
7	37-38	Minutes portion of azimuth or right ascension.
8	39-43	Seconds portion of azimuth or right ascension. The decimal point is assumed to be between columns 40 and 41.

Field	Columns	Contents-Description
9	44-46	Degrees portion of elevation or declination.
10	47-48	Minutes portion of elevation or declination.
11	49-52	Seconds portion of elevation or declination. The decimal point is assumed to be between columns 50 and 51.
12	56	Type of observation 0, right ascension and declination 1 or 3, azimuth, elevation and possibly range
13	57	Code for equinox of reference 1, equinox of 1855.0 2, equinox of 1875.0 3, equinox of 1900.0 4, equinox of 1950.0
14	59-63	Range in kilometers. The decimal point is assumed to be between columns 59 and 60.
15	64	Power of ten by which to multiply range field to obtain true range.

The following comments will be useful to the reader in preparing a Smithsonian format observation tape.

Each observation is read according to the following FORMAT statement.

```
FORMAT (13X,I4,F6.0,2F2.0,F6.4,F3.0,F2.0,F5.3,F3.0,F2.0,F4.2,3X,2I1,1X,F5.4,I1)
```

When all desired observations have been prepared on cards, the cards are put on a tape in ascending chronological sequence with a blank card following the last observation.

A prefix of 1900 is assumed for the year field of each observation.

The equinox reference code (field 13) is only interpreted for right ascension, declination type observations, while the range field is only interpreted for azimuth, elevation type observations. An azimuth, elevation type observation is converted to a range, azimuth, elevation type observation if the range field is greater than zero.

Since SAO uses the same format for field reduced and photo-reduced observations, it is possible to process either type observation, but not both together. This is true since different time systems are used for field reduced and photo-reduced observations. Since the program does not determine the type of instrumentation used in the observation, the user is required to separate the types manually. This can be done by sorting the observation cards on columns 8 which will contain a 7 for all photo-reduced observations.

A.1.2 AFCRL Format

Each observation is punched onto a card which contains the following information. (This description will also be limited to the portion of the card interpreted by the program.)

Field	Columns	Contents-Description
1	9-12	Station number.
2	14	Units digit of year of observation
3	16-17	Month of observation.
4	19-20	Day of observation,
5	22-23	Hour of observation.
6	25-26	Minute of observation.
7	28-32	Seconds of observation. The decimal point is assumed to be between columns 29 and 30.
8	34-39	Elevation in degrees. The decimal point is assumed to be between columns 36 and 37.
9	41-46	Azimuth in degrees. The decimal point is assumed to be between columns 43 and 44.
10	48-56	Range in kilometers. The decimal point is assumed to be between columns 53 and 54.

The following comments will be useful to the reader in preparing an AFCRL format observation tape.

Each observation is read according to the following FORMAT statement:

FORMAT (8X, I4, 1X, F1.0, 4(1X, F2.0), 1X, F5.3, 2(1X, F6.3), 1X, F9.3)

When all desired observations have been prepared on cards, the cards are put on a tape in ascending chronological sequence with a blank card following the last observation.

A prefix of 1960 is assumed for the year field of the observation.

A.1.3 SPADATS Format

Each observation is punched onto a card which contains the following information. (This description will also be limited to the portion of the card interpreted by the program. A complete description may be found in reference 10.)

Field	Columns	Contents-Description
1	1	Classification indicator. S=Secret, C=Confidential, U=Unclassified.
2	7-9	Station number.
3	10-11	Last two digits of year of observation.
4	12-14	Day of year of observation.
5	15-16	Hour of observation.
6	17-18	Minute of observation.
7	19-20	Second of observation.
8	21-23	Thousandths of a second.
9	24	Tens digit of elevation/declination field in degrees. This field may also be overpunched with a minus punch.
10	25-29	Remainder of elevation/declination field in degrees.
11	31-32	Hundreds and tens digit of azimuth field or hours portion of right ascension field. Azimuth measurements are in degrees.
12	33-34	Units and tenths digits of azimuth field or minutes portion of right ascension field.
13	35-37	Hundredths, thousandths, and ten thousandths digits of azimuth field or tens, units and tenths digits of seconds portion of right ascension field.

Field	Columns	Contents - Description
14	39-45	Range field in kilometers. The decimal point is assumed to be between columns 40 and 41.
15	46	Power of ten by which to multiply field 14 to obtain true range.
16	75	Type of observation 1 - azimuth, elevation 2, 3, 4 - range, azimuth, elevation 5 - right ascension, declination 0, 6, 7, 8, 9 - not recognized
17	76	Equinox indicator for right ascension, declination observations 1 - Equinox of 1900.0 2 - Equinox of 1920.0 3 - Equinox of 1950.0 4 - Equinox of 1975.0 5 - Equinox of 2000.0 6 - Equinox of 1850.0 7 - Equinox of 1855.0 8 - Equinox of 1875.0 9 - Equinox of 1960.0 0 - Not recognized

The following comments will be useful to the reader in preparing a SPADATS format observation tape.

Each observation is read according to the following FORMAT statement.

```
FORMAT (A1,5X,I3,I2,I3,2F2.0,F5.3,A1,F5.4,1X,2F2.0,F3.1,1X,F7.5,I1,28X,2I1)
```

When all desired observations have been prepared on cards, the cards are put on a tape in ascending chronological sequence with a blank card following the last observation.

A prefix of 1900 is assumed for the year field of the observation.

A.2 Station and Equinox Data Preparation

The station data tape is generated by an off-line card-to-tape operation. The tape includes three sections of data, each of which must be followed by a blank card.

Section one consists of a list of classified stations along with their unclassified codes which are used in all output which references these stations. Each card of section one contains two numbers, where the first number is the classified number of a station, and the second number is its unclassified code. This type card is punched in accordance with the following FORMAT statement:

FORMAT (2I5)

Notice that if no classified station data exists, the first record on the station tape will be a blank card.

The second section of data consists of one card for each station for which there is an observation. Each card contains the station number, latitude (in degrees), longitude (in degrees), and height above sea level (in meters) which are punched in the above specified order in accordance with the following FORMAT statement:

FORMAT (I4, F7.4, F8.4, F6.0)

Latitudes may range from -90 to +90 with the northern hemisphere assuming positive latitude values. Longitudes may range from -360 to +360 with longitudes which are west of Greenwich being considered positive.

The format for station data cards is in full agreement with the station deck which may be obtained from Space Track. The distributed Smithsonian station data deck is in a different format, however, and a program is required to make the necessary conversion. This program has been written and is available along with a "converted" Smithsonian station deck which was obtained from this program.

The third section of the station tape contains equinox correction data which are used in updating right ascension (α) and declination (δ) measurements from a given equinox to a common equinox. An equinox correction is computed from the following formulas.[4]

Let

$$A = \alpha + \frac{\Delta t}{2} \left(C_1 + C_2 \tan \delta \sin \alpha \right)$$

and

$$D = \delta + \frac{\Delta t}{2} C_2 \cos \alpha$$

Then the updated values for α and δ are given by

$$\alpha_U = \alpha + \Delta t \left(C_3 + C_4 \tan D \sin A \right)$$

and

$$\delta_U = \delta + \Delta t \, C_4 \cos A$$

where

Δt = the difference in time between the desired equinox and a given equinox

C_1, C_2, C_3, C_4 are the appropriate precessional constants.

The information associated with each given equinox is punched on a card which contains the year of the equinox, C_1, C_2, C_3, and C_4 in the specified order, in accordance with the following FORMAT statement:

FORMAT (5F12.0)

The order of the equinox cards is important since the routines which read observational data refer to the equinox cards by number rather than by year of equinox. Consequently, the present version of the program requires that the first nine equinox cards on the station tape correspond to the equinoxes of 1855, 1875, 1900, 1950, 1850, 1920, 1960, 1975, 2000. These nine cards presently contain the following values:

	Year	C_1	C_2	C_3	C_4
1	1855.0	.0127979	.0055696	.0128022	.0055683
2	1875.0	.0127995	.0055691	.0128029	.0055681
3	1900.0	.0128014	.0055686	.0128039	.0055678
4	1950.0	.0128053	.0055674	.0128058	.0055672
5	1850.0	.0127975	.0055697	.0128020	.0055684
6	1920.0	.0128029	.0055681	.0128047	.0055676
7	1960.0	.0128060	.0055671	.0128062	.0055671
8	1975.0	.0128072	.0055668	.0128068	.0055669
9	2000.0	.0128091	.0055662	.0128078	.0055666

Because of storage limitations, there is a maximum number of cards which may be present in each of the three sections of the station tape. The present version of the program allows for twenty-five classified stations in section one, one hundred stations in section two, and ten equinoxes in section three. In the event a station list contains more than the allowable number of entries, it will be necessary to isolate the portion of the station list referred to by the observation tape. For this purpose, a preprocessing program has been written which reads observations and lists all stations referred to by these observations. (This program performs two additional functions. The observation tape is checked for chronological correctness and for the validity of numeric fields.)

A.3 Density Model Preparation

The program currently uses Jacchia's model atmosphere.[5] This model atmosphere is contained on 534 cards which give densities for heights between 120 and 1000 kilometers in steps of 10 kilometers and for temperatures between 2400 and 600 degrees Kelvin in steps of 50 degrees. The routine which reads this model (DENS) has been made quite flexible by the addition of one card at the beginning of the model which lists various parameters of the model. This capability allows future expansions or contractions of the model without requiring any revisions to the DENS routine. The additional card contains the following information.

Field	Value	Description
1	37	Number of temperature values.
2	2400	Initial temperature value.
3	-50	Increment in temperature entries.
4	100	Allowable extrapolation (below end of model) for low temperatures.
5	89	Number of height values.
6	120	Initial height value.
7	10	Increment in height entries.
8	50	Factor used in high height extrapolation.

This card is read according to the following FORMAT statement.

FORMAT (2 (I6, 3F6.0))

The model itself contains seven entries per card with density values for all temperatures at a given height requiring six cards. (The sixth card contains only two entries.) The density values are recorded as common logarithms.

The model is read according to the following FORMAT statement.

FORMAT (7F10.0)

The density tape required by the program is prepared by placing on a tape the special card (described above) followed by the 534 cards of the model.

A.4 Specification Cards

This section describes the method of preparing the cards which specify the parameters of a run.

There are four types of cards which may be used. These are now described.

A.4.1 Remark Card

This type of card is used to introduce any desired information into the output of a run. Remark cards if present are the first cards of a run. A remark card is identified by the word REMARK in columns 1 - 6, with the remainder of the card being arbitrary.

A.4.2 Starting Elements Cards

Following any remark cards is a set of two cards which contains the initial or estimated orbital parameters of the satellite at a specified time. The specified time should be near the beginning of the period of time to be processed. The first card of this set is prepared as follows:

Field	Columns	Contents-Description
1	18-19	Two low order digits of year of elements.
2	21-22	Month of elements.
3	24-25	Day of elements.
4	27-28	Hour of elements.
5	30-31	Minute of elements.
6	33-38	Second of elements.
7	53-58	Nighttime exospheric temperature at time specified in fields 1 - 6.
8	65-72	Area/mass ratio (cm^2/ g)

This card is read according to the following FORMAT statement.

FORMAT (16X, 5F3.0, F7.3, 14X, F6.0, 6X, F8.5)

The second card is prepared as follows:

Field	Columns	Contents - Description
1	1-12	Mean anomaly in degrees.
2	13-24	Argument of perigee in degrees.
3	25-36	Right ascension of ascending node in degrees.
4	37-48	Inclination angle in degrees.
5	49-60	Eccentricity.
6	61-72	Semi-major axis in kilometers.

All angular elements should be reduced to the interval (0, 360) with the additional restriction that the inclination angle be limited to the interval (0, 180).

This card is read according to the following FORMAT statement.

FORMAT (4F12.4, F12.7, F12.3)

A typical set of starting element cards might be

```
TIME OF ELEMENTS 65 01 02 03 04 05.678 NIGHT TEMP. 678.  A/M .0123
       123.45      123.45     123.45     12.345      .00345   6543.2
```

Notice that the first card is made quite "readable" by the addition of text in uninterpreted portions of the card.

A.4.3 Change Card

At the beginning of each run, the program assigns values to a number of parameters which are not likely to change from run to run. The value of any of these parameters may be modified by the user by means of a change card. The contents of the card are as follows:

Field	Columns	Contents - Description
1	1-6	CHANGE
2	17-19	Number of parameter to be modified.
3	20	Indication of parameter mode.
4	24-37	Modified value of the parameter.

If field three is blank, the program assumes that the parameter to be changed is a real variable; otherwise the program assumes that the parameter to be changed is an integer variable.

The change card is read according to the following FORMAT statement.

FORMAT (A6, 10X, I 3, A1, 3X, E14.7)

Notice that field four must be in the real mode regardless of the mode of the parameter to be changed.

A typical change card might be:

CHANGE PARAMETER 3* TO 10.0E + 00

which informs the program that the observation tape to be processed is to be found on logical tape number 10.

A complete description of the parameter array can be found in paragraph A.5 of this appendix.

Change cards if present come immediately after the starting element cards.

A.4.4 Run Card

Following any change cards is a run card which is the last card of each run. The run card is prepared as follows:

Field	Columns		Contents-Description
1	10-11	Start time of processing	Two low order digits of year
2	13-14		Month
3	16-17		Day
4	19-20		Hour
5	22-23		Minute
6	28-29	End time of processing	Two low order digits of year
7	31-32		Month
8	34-35		Day
9	37-38		Hour
10	40-41		Minute

Field	Columns	Contents-Description
11	56	Observation format indicator 1, Smithsonian format 2, AFCRL format 3, SPADATS format
12	72	Atmospheric model indicator 0, 1, 3-9, Jacchia model 2, Drag model (inactive)

The run card is read according to the following FORMAT statement.

FORMAT (8X, 5F3.0, 3X, 5F3.0, 13X, I 2, 14X, I2)

A typical run card might be:

```
RUN FROM 69 01 02 00 00 TO 69 01 03 00 00 FOR OB. TYPE 3 AND MOD. TYPE 1
```

which requests the program to process all observations on the tape containing SPADATS format observations which were recorded on January 2, 1969 using the Jacchia model atmosphere.

A. 4. 5 End Card

The last run in a sequence of CADNIP runs must be followed by a card containing the letters END in columns 1-3 and blanks in columns 4-6.

A. 5 Description of Parameter Table

The parameter table contains parameters whose values are subject to change by the user by means of the change card as described in paragraph A. 4. 3 of this appendix. As the parameter table is completely initialized by the program at the beginning of each run, the effect of CHANGE cards is not carried over from run to run.

Since the user is required to know the mode of any parameter he desires to modify, the following convention is adopted. A parameter is a real parameter if and only if its value is printed in this report as a number containing a decimal point.

Parameter Number	Assigned Value	Parameter Use
1	1	Logical number of station and equinox tape.
2	2	Logical number of atmospheric model tape.
3	3	Logical numberof observation tape.
6	6378.165	Equatorial radius of earth in kilometers.
7	1.0/298.3	Flattening of earth.
8	806.8136	Canonical unit of time in seconds.
9-10	690101.0,0.0	The date 1 Jan. 1969 in internal format.
11	.279078160	Sideral angle of Greenwich at 1 Jan. 1969 in revolutions.
12	.27379093E-2	Modified value for rotation rate of Greenwich in revolutions per day.
14-20	0.0	Lunar orbital elements at a specified time.
21	0.0	Ratio of mass of moon to mass of earth.
22	2.2	Coefficent of drag (C_D).
23	1.0	Ratio of rotation rate of atmosphere to rotation rate of earth.
24	1.0	Initial factor to apply to atmospheric model.
25	0.1	Smallest allowable factor to apply to model.
26	10.0	Largest allowable factor to apply to model.
27	.28	R — 6 constants appearing in Jacchia's model for the diurnal variation.
28	1.5	m
29	2.5	n
30	-45.0	β
31	12.0	p
32	45.0	γ
33	0.0	Factors controlling assumed latitude of the center of the diurnal atmospheric bulge.
34	0.0	
35	0.125	Desired accuracy of solar position (in days).
37	15.0	Runge-Kutta time step in seconds for regular integration
38	4	Ratio of predictor-corrector time step to Runge-Kutta time step for regular integration.
40	0.375	Desired time span in days of a set of observations.

Parameter Number	Assigned Value	Parameter Use
41	0.125	Amount of days by which to modify time span of a set of observations if necessary.
42	0.25	Minimum allowable time span in days of a set of observations.
43	2.0	Maximum allowable time span in days of a set of observations.
46	6	Minimum number of observations necessary in preliminary adjustment procedure.
47	1	Number of iterations in preliminary adjustment procedure.
48	3.0	The product of this parameter and sigma determine tolerance used in rejection of observations during preliminary adjustment procedure.
51	7	Minimum number of observations necessary in differential correction procedure.
52	0	Parameter controlling intermediate output from differential correction procedure.
53	14	Minimum number of equations of condition in differential correction procedure.
54	10	Maximum allowable number of iterations in differential correction procedure.
55	2	Number of iterations in differential correction procedure in which to compute partial derivatives.
56	15.0	R.K. time step in seconds for computing partial derivatives of present position with respect to initial position and velocity.
57	15.0	R.K. time step in seconds for computing partial derivatives of present position with respect to density.
58	4	Ratio of predictor-corrector time step to R.K. time step for computing partial derivatives of present position with respect to initial position and velocity.
59	4	Ratio of predictor-corrector time step to R.K. time step for computing partial derivatives of present position with respect to density.

Parameter Number	Assigned Value	Parameter Use
61	1.0E-6	Increment used in estimating partial derivatives of present position with respect to initial position and velocity.
62	1.0E-3	Increment used in estimating partial derivatives of present position with respect to density.
65	10	Number of iterations in differential correction procedure in which to reject bad observations.
66-67	5.0	Tolerances in degrees used for rejection of right ascension/azimuth and declination/elevation measurements during first iteration of differential correction procedure.
68	500.0	Tolerance in kilometers for rejection of range measurements during first iteration of differential correction procedure.
69	50.0	Tolerance in kilometers for rejection of range measurements during second and subsequent iterations of differential correction procedure.
70	3.0	The product of this parameter and sigma determines tolerance used in rejection of right ascension/azimuth and declination/elevation measurements during second and subsequent iterations of the differential correction procedure.
71	5.0	Tolerance used to determine whether differential correction procedure has diverged.
72	.01	Tolerance used to determine desired convergence in differential correction procedure.
73	.1	Tolerance used to determine acceptable convergence in differential correction procedure.
74	15.0	Tolerance in minutes of arc used to determine acceptability of the differential correction results.
75	.1	Tolerance used to determine acceptability of the final density.
76	xxxx.	Equinox to which observations are updated.
77	xxx.	Standard height density.

Parameter Number	Assigned Value	Parameter Use
79	4	Order of gravity model used in preliminary calculations.
80	8	Order of gravity model used in calculations requiring full accuracy.
81	See Table A - 1	$C_2^0 = -J_2$
82-83		C_2^1, C_2^2
84		$C_3^0 = -J_3$
85-87		C_3^1, C_3^2, C_3^3
88		$C_4^0 = -J_4$
89-308		$C_4^1, C_4^2, C_4^3, C_4^4, C_5^1 \ldots, C_{20}^{20}$
309-517		$S_2^1, S_2^2, S_3^1, S_3^2, \ldots, S_{20}^{20}$

Table A - 1

Values of Zonal, Tesseral, and Sectorial Harmonics

n \ m	0	1	2	3	4	5	6	7	8
2	-1082.645E-06		1.536E-06						
3	2.546E-06	2.091E-06	.251E-06	.782E-07					
4	1.649E-06	-.543E-06	.738E-07	.509E-07	-.112E-08				
5	.210E-06	-.677E-07	.102E-06	-.172E-07	-.206E-08	.384E-09			
6	-.646E-06	-.370E-07	.858E-08	-.112E-08	-.167E-09	-.253E-09	-.932E-11		
7	.333E-06	.144E-06	.363E-07	.352E-08	-.323E-09	.269E-10	-.145E-10	.102E-11	
8	.270E-06	-.515E-07	.213E-08	-.374E-09	-.277E-09	-.959E-11	-.475E-12	-.444E-13	-.316E-12

$C_{n,m}$

n \ m	0	1	2	3	4	5	6	7	8
2			-8.721E-07						
3		.287E-06	-.184E-06	2.259E-07					
4		-.445E-06	.148E-06	-.114E-07	.486E-08				
5		-.882E-07	-.375E-07	.231E-09	.498E-09	-.146E-08			
6		-.212E-07	-.455E-07	.643E-09	-.196E-08	-.370E-09	-.361E-10		
7		.114E-06	.162E-07	.254E-09	-.217E-09	.191E-10	.437E-11	.178E-11	
8		.447E-07	.320E-08	.404E-10	-.157E-10	.214E-10	.888E-11	.158E-12	.130E-12

$S_{n,m}$

Parameters 9-12 allow the user to specify an initial position and a rotation rate for Greenwich at a given time. Parameter 9 contains a time in the form of DAY + 100 X MONTH + 10000 X YEAR. Parameter 10 contains a fraction of a day. Parameter 11 specifies the sidereal angle of Greenwich at the time specified by parameters 9 and 10. Parameter 12 is a modified value for the rotation rate of Greenwich. The modified value which is essentially $\dot{\theta}$ - 1.0, allows for the retention of sufficient accuracy without resorting to double precision arithmetic.

Lunar effects are currently ignored by the program. Consequently, parameters 14-21 are set to zero.

Parameter 24 will be modified by the preliminary adjustment procedure and the differential correction procedure. Parameters 25 and 26 act as lower and upper bounds on parameter 24. This feature is useful in controlling computational difficulties which may arise because of grossly inaccurate starting conditions or insufficient drag effects.

In the special case where parameters 25 and 26 are both set equal to 1.0, the preliminary adjustment procedure will correct the mean anomaly and semi-major axis only, while the differential correction procedure will be limited to an improvement of the 6 Keplerian elements. This mode of operation will result in a significant saving of computer time, since the partial derivatives of present position with respect to density are unnecessary.

Parameters 33 and 34 allow the user to specify a linear function for the assumed center of the diurnal atmospheric bulge as follows:

bulge center = parameter 33 + parameter 34 times $\delta_{\odot}$

where $\delta_{\odot}$ = declination of the sun.

Parameter 33 is assumed to be in degrees while parameter 34 is a pure number.

The solar position is only recomputed when the difference between the present time and the time at the previous computation of solar position is more than 3 hours as indicated by parameter 35.

The program initially attempts to process epochs which span 9 hours as specified by parameter 40. If unsuccessful, the epoch length is successively increased by an amount specified by parameter 41 until the program succeeds in processing an epoch or the maximum allowable epoch (as specified by parameter 43) is exceeded. Note that an unreasonably small value for parameter 40 will result in a considerable waste of computer time. Factors to be considered in choosing an appropriate value for parameter 40 are the area-to-mass ratio and (perigee) height of the satellite, the observation quality, and the desired accuracy of the density result.

The mean anomaly residual calculation (which is used by the preliminary adjustment procedure) may at times give slightly inaccurate results. This deficiency may be overcome by an iterative procedure. Hence, the need for parameter 47. This parameter is automatically increased by 1 for the first epoch to be processed and for any epoch which follows an unsuccessful differential correction attempt. The entire preliminary adjustment procedure may be omitted by setting parameter 47 to -1.

If parameter 52 is greater than zero, the results of each iteration of the differential correction procedure will be printed. Otherwise, a printout will occur only during the final iteration.

Parameter 55 is automatically decreased by 1 for any epoch in which the preliminary adjustment procedure has operated successfully.

The differential correction procedure is considered to have diverged, if on a given iteration the value of sigma (standard error of the correction) is five times the sigma of the previous iteration as indicated by parameter 71.

The differential correction procedure is considered to have converged if 2 successive sigmas agree to 1% as indicated by parameter 72. If the procedure reaches the last allowable iteration without diverging or converging, the procedure is considered to have converged if the last 2 sigmas agree to 10% as indicated by parameter 73.

Parameter 76 is used by the program for updating right ascension and declination measurements from a given equinox to a common equinox. The program initially sets parameter 76 equal to January 1 of the year closest to the time of the starting elements as specified on the starting elements cards.

Parameter 77 is set by the program to a height (to the nearest ten kilometers) close to perigee as specified on the starting elements cards.

Parameter 76 and 77 may be modified by the user in the usual manner.

The parameter table contains all zonal, tesseral, and sectorial harmonics through 8, 8. Values shown in Table A - 1 may be modified by means of the following two formulas which give the correspondence between a parameter number and a given harmonic.

C_n^m is placed in parameter number $[80 + \frac{(n)(n+1)}{2} + m-2]$

S_n^m is placed in parameter number $[308 + \frac{(n-1)(n)}{2} + m - 1]$

All harmonic coefficients are assumed to be unnormalized.

The present version of the LGNDR routine which evaluates the associated Legendre functions contains all functions up through (8, 8). Additional functions up through (20, 20) are available and may be easily inserted into the LGNDR deck. C_n^m or S_n^m requires P_n^m and P_n^{m+1} for $m < n$ and P_n^m for $m = n$.

A. 6 DRIVE Routine

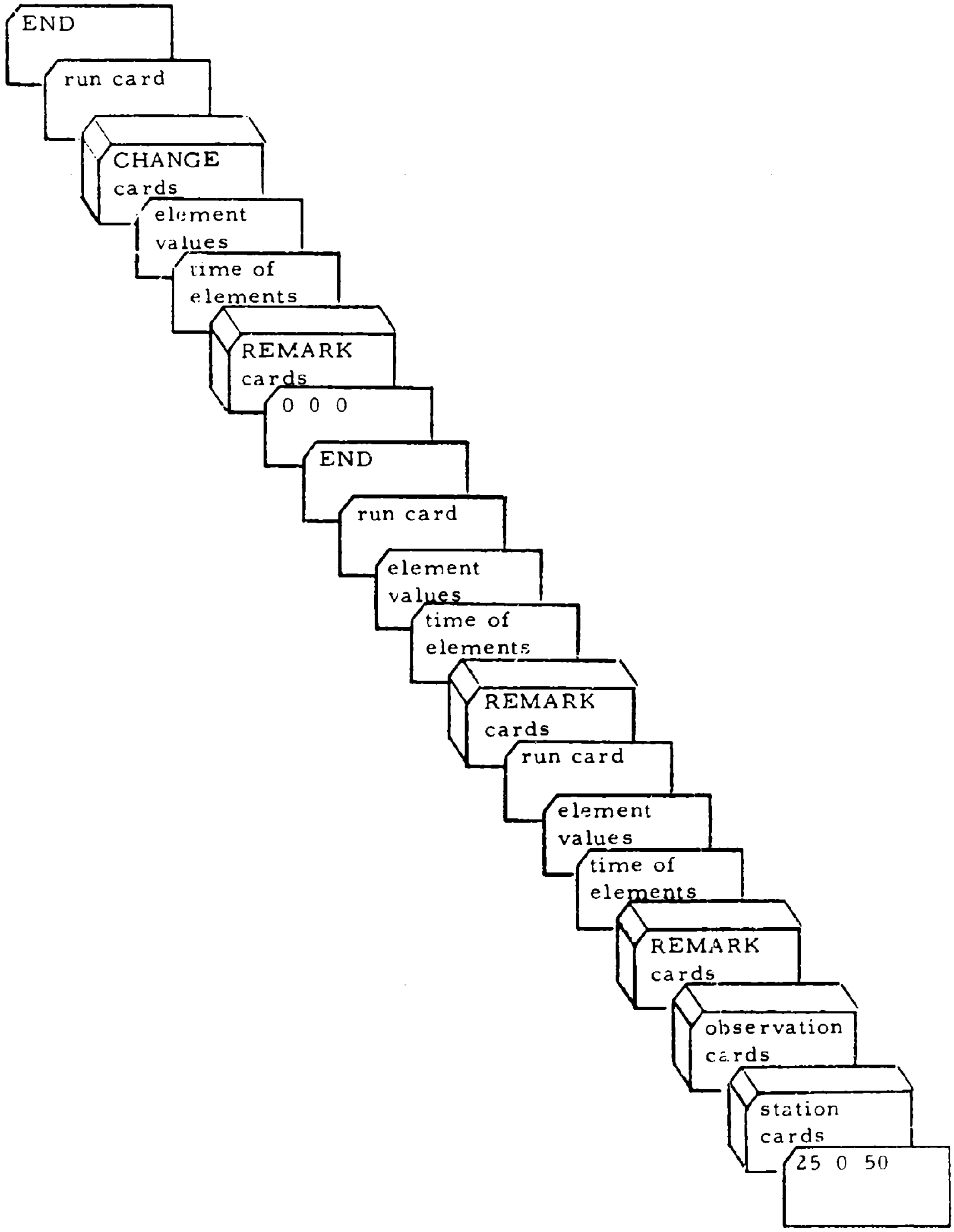
The DRIVE routine calls in the main control routine ADMON, which in turn activates lower level routines as described in section 4. The logical numbers of the system input and output tapes, namely 5 and 6 respectively, are transmitted by DRIVE as calling sequence arguments to ADMON. An additional degree of flexibility has been incorporated into the DRIVE program which allows the user to generate any of the three required data tapes at run time. The use of this feature is described in paragraph A. 7 of this appendix.

A. 7 Data Deck Setup for Running CADNIP

Figure A-1 shows a typical data deck setup for three CADNIP runs.

Figure A-1

Sample Data Deck



In this figure, capital letters are used for cards whose type is determined by a specific code as described in paragraph A. 4 of this appendix. The types of all other cards appearing in this figure are determined by their relative positions in the data deck.

The first card shown in figure A. 1 is read in by the DRIVE routine. This card contains three numbers which are read according to the following FORMAT statement.

FORMAT (3I4)

The three numbers on this card indicate respectively whether the required station, density, and observation tapes are available from a previous operation, or are to be generated at run time. Thus in the example of figure A-1, the user intends to generate at run time the station and observation tapes, while a density tape is available from a previous operation. The logical numbers of tapes generated at run time are defined by DRIVE to be the same as those assigned by ADMON in the parameter table, namely 1, 2 and 3. Thus, in this example 25 cards will be copied onto logical tape 1 and the next 50 cards will be copied onto logical tape 3. Cards used to generate data tapes in this fashion must be identical in form and in order to cards used in the preparation of off-line data tapes. Thus the 25 station cards must include all cards which comprise the three sections of a station tape along with any required blank cards as described in paragraph A. 2 of this appendix. Similarly, the blank card which follows the last observation on an observation tape will be the last card of the 50 observation cards.

After generating the required data tapes, the DRIVE routine calls ADMON which in turn reads and processes all cards up to and including the first run card. When the time interval appearing on this run card has been fully processed, ADMON reads and processes the next set of cards up to and including the second run card.

Since no CHANGE cards (which may alter the logical numbers of data tapes to be processed) appeared in either of the first two CADNIP runs, the same data tapes will be used for both runs. Thus it is possible to use run time generated tapes in a number of runs.

The END card which is then read by ADMON effectively returns control to the DRIVE routine. At this point, DRIVE reads the card containing all zeros (indicating that no data tapes are to be generated) and immediately calls ADMON. ADMON then reads and processes the third CADNIP run, and upon encountering the last END card again returns control to DRIVE. The entire procedure is terminated by an end-of-file condition encountered by DRIVE.

It will be observed that the first END card and the following card containing all zeros which appear in figure A-1 are unnecessary and could have been omitted, in which case all three runs would have been processed in succession by ADMON without returning control to DRIVE after the second run. Note, however, that it would be improper to remove only one of these two cards.

Appendix B

DESCRIPTION OF NUMERICAL INTEGRATION TECHNIQUE

Appendix B

Introduction

Satellite positions are predicted in the CADNIP program by a numerical integration of the equations of motion in rectangular coordinates, expressed in the form of six simultaneous first-order equations. The integration algorithm is a predictor-corrector method, using a sixth-order Adams-Bashforth predictor, a sixth-order Adams-Moulton corrector (which is applied only once), and modifiers of so-called Hamming type to improve both predictor and corrector. Starting values are generated by the classical fourth-order Runge-Kutta method.

Stability, accuracy, and simplicity were the principal criteria considered in choosing the integration method to be implemented in CADNIP. For a discussion of the relative merits of the various approac s to the numerical integration of satellite trajectories, the reader is referred to Conte.[11]

For the sake of notational uniformity, the equations of motion will be expressed in vector form as

$$\dot{Y} = F(Y,t) \tag{B.1}$$

and subscripts will be used to designate values at particular instants of time, e.g.

$$Y_i = Y(t_i) \quad \text{and} \quad F_i = F(Y_i, t_i)$$

1. The Runge-Kutta Starter

The multistep predictor-corrector method used in CADNIP requires six previous values of the right-hand sides, F, and of the state vector, Y, in order to generate one new step of the solution. At the beginning of an integration, these starting values are generated by a self-starting single-step method, the classical fourth-order Runge-Kutta algorithm. (See, for example, Hildebrand[12], p. 237; Henrici[13], pp. 121-122.)

There is a provision in CADNIP for the use of three different step sizes, depending on the purpose for which the numerical integration is being done:

- ephemeris generation
- evaluation of partial derivatives with respect to initial state
- evaluation of partial derivatives with respect to density

All three of these parameters are currently set at 15 seconds. Similarly, the ratio between the Runge-Kutta step size and the step size to be used by the predictor -corrector algorithm may also be chosen individually for each of the three cases listed above; at present, however, each of these three ratios is set at 4. Thus a total of 20 Runge-Kutta steps are taken in order to generate the required set of starting values.

The reader may consult section A.5 for information on changing these parameters.

2. The Predictor-Corrector Method

The basic algorithm consists of four steps:

- Predict
- Modify predictor
- Correct
- Modify corrector

at the end of which a new state vector has been computed. The basic formulas are derived in sections 2.1 and 2.2, below; a concise summary and flow chart are presented in section 2.3.

2.1 Basic Predictor-Corrector Formulas

For the sake of simplicity, the formulas are derived here in scalar form. The extension to vector variables is immediate. References 12, 13 and 14 may be consulted; however, only reference 14 gives the explicit form of formula (B.5) correctly.

2.1.1 The Predictor

We write $y_{n+1} = y_n + \Delta y$, where Δy represents the change in y between the times $t = t_n$ and $t = t_{n+1}$. That is,

$$\Delta y = \int_{t_n}^{t_{n+1}} \left(\frac{dy}{dt}\right) dt$$

Since the differential equation is of the form

$$\frac{dy}{dt} = f(y, t)$$

it follows that

$$\Delta y = \int_{t_n}^{t_{n+1}} f(y,t)\, dt \quad \text{where } t_{n+1} = t_n + h \tag{B.2}$$

Of the many forms for representing a collocation polynomial approximating $f(y, t)$, we choose one employing backward differences:

$$\nabla f_n = f_n - f_{n-1}$$

$$\nabla^2 f_n = \nabla(\nabla f_n) = \nabla f_n - \nabla f_{n-1} = f_n - f_{n-1} - (f_{n-1} - f_{n-2})$$

$$= f_n - 2f_{n-1} + f_{n-2}$$

etc.

In terms of these backward differences, the polynomial $P(t)$, agreeing with $f(y, t)$ at the points $t_n, t_{n-1}, \ldots, t_{n-k}$,

can be written in the form:

$$P(t) = f_n + s\nabla f_n + \frac{s(s+1)}{2!}\nabla^2 f_n + \ldots + \frac{s(s+1)\ldots(s+k-1)}{k!}\nabla^k f_n \tag{B.3}$$

$$\text{where} \quad s = \frac{t - t_n}{h} \quad \text{or} \quad t = t_n + sh$$

Then $\displaystyle\int_{t_n}^{t_{n+1}} P(t)\, dt = \int_0^1 \left[f_n + s\nabla f_n + \ldots\right](h\, ds)$

or

$$\int_{t_n}^{t_{n+1}} P(t)dt = h \sum_{\alpha=0}^{k} C_\alpha \nabla^\alpha f_n$$

where

$$C_o = \int_0^1 ds = 1$$

$$C_1 = \int_0^1 s ds = \frac{1}{2}$$

$$C_2 = \int_0^1 \frac{s(s+1)}{2!} ds = \frac{5}{12}$$

$$C_3 = \int_0^1 \frac{s(s+1)(s+2)}{3!} ds = \frac{3}{8}$$

$$C_4 = \int_0^1 \frac{s(s+1)(s+2)(s+3)}{4!} ds = \frac{251}{720}$$

$$C_5 = \int_0^1 \frac{s(s+1)(s+2)(s+4)}{5!} = \frac{95}{288}$$

etc.

The final form is

$$y_{n+1} = y_n + h(f_n + \frac{1}{2}\nabla f_n + \frac{5}{12}\nabla^2 f_n + \frac{3}{8}\nabla^3 f_n + \frac{251}{720}\nabla^4 f_n + \frac{95}{288}\nabla^5 f_n)$$

$$+ \text{ Remainder term} \qquad \text{(B.4)}$$

if fifth differences are retained. This is the Adams - Bashforth formula in the backward-difference form.

To transform from backward difference to ordinate form, we use the relations

$$\nabla^k f_n = f_n - \binom{k}{1} f_{n-1} + \binom{k}{2} f_{n-2} - \ldots + (-1)^k f_{n-k}$$

Considering, in particular, formula (B.4)

$$y_{n+1} = \left[y_n + h(f_n + \ldots + \frac{95}{288}\nabla^5 f_n) \right] + \text{ Remainder}$$

we obtain, upon carrying out the substitutions and collecting terms,

$$y_{n+1} = y_n + \frac{h}{1440}(4277 f_n - 7923 f_{n-1} + 9982 f_{n-2} - 7298 f_{n-3} + 2877 f_{n-4}$$

$$- 475 f_{n-5}) + \text{Remainder} \qquad (B.5)$$

This is the predictor formula used in CADNIP.

2.1.2 The Corrector

Another way of defining a collocation polynomial P(t) to approximate f(y, t) would be to require agreement at the points t_{n+1}, t_n, ..., t_{n-k+1}. P(t) would then be of the form

$$P^*(t) = f_{n+1} + s^* \nabla f_{n+1} \frac{s^*(s^*+1)}{2!} \nabla^2 f_{n+1} + \dots \qquad (B.6)$$

where $s^* = \dfrac{t - t_{n+1}}{h}$ or $t = t_{n+1} + s^* h$

Now $\displaystyle\int_{t_n}^{t_{n+1}} P^*(t)dt = \int_{-1}^{0} \left[f_{n+1} + s^* \nabla f_{n+1} + \dots \right] (h\, ds^*)$

$$\int_{t_n}^{t_{n+1}} P^*(t)dt = h \sum_{\alpha=0}^{k} C_\alpha^* \nabla^\alpha f_{n+1}$$

where

$$C_0^* = 1$$

$$C_1^* = \int_{-1}^{0} s^* ds^* = -\frac{1}{2}$$

$$C_2^* = \int_{-1}^{0} \frac{s^*(s^*+1)}{2!} ds^* = -\frac{1}{12}$$

$$C_3^* = \int_{-1}^{0} \frac{s^*(s^*+1)(s^*+2)}{3!} ds^* = -\frac{1}{24}$$

etc.

Retaining fifth differences, as before, we obtain the formula

$$y_{n+1} = y_n + h\left(f_{n+1} - \frac{1}{2}\nabla f_{n+1} - \frac{1}{12}\nabla^2 f_{n+1} - \frac{1}{24}\nabla^3 f_{n+1} - \frac{19}{720}\nabla^4 f_{n+1} - \frac{3}{160}\nabla^5 f_{n+1}\right)$$

$$+ \text{ Remainder term} \qquad \text{(B.7)}$$

which is the Adams-Moulton formula in backward-difference form. Since (B.7) expresses y_{n+1} in terms of f_{n+1}, which is itself a function of y_{n+1}, the formula is <u>implicit</u>. It must be used in conjunction with a predictor of explicit type (e.g. formula B.4) which supplies a value of f_{n+1}.

The transformation to ordinate form is carried out as before, with the result

$$y_{n+1} = y_n + \frac{h}{1440}(475 f_{n+1} + 1427 f_n - 798 f_{n-1} + 482 f_{n-2} - 173 f_{n-3} + 27 f_{n-4})$$

$$+ \text{ Remainder term} \qquad \text{(B.8)}$$

This is the corrector formula used in CADNIP.

2.1.3 The Remainder Terms

The local quadrature error introduced by using a finite number of differences has as its principal part the first neglected term. Thus, for the predictor formula,

$$y_{n+1} = \left[y_n + h(f_n + \ldots + \frac{95}{288}\nabla^5 f_n)\right] + h\left(\frac{19087}{60480}\right)\nabla^6 f_n \qquad \text{as } h \longrightarrow 0$$

For any n, the backward difference may be expressed in the form

$$\nabla^n f = h^n \frac{d^n}{dt^n} f + O(h^{n+1})$$

Thus $$h\nabla^6 f_n = h^7 \frac{d^6}{dt^6} f(t_n)$$

Since $$\frac{dy}{dt} = f$$

$$h\nabla^6 f_n = h^7 y^{[7]}$$

Therefore the principal error term for the predictor is

$$\text{Remainder} = +\frac{19087}{60480} h^7 y^{[7]} \tag{B.9}$$

The principal error term for the corrector is derived in exactly the same way

$$\text{Remainder} = -\frac{863}{60480} h^7 y^{[7]} \tag{B.10}$$

2.2 The Modifiers

If y_{i+1} designates the true value of $y(t_{i+1})$ then, in the limit as $h \to 0$,

$$y_{i+1} = y_{i+1}^{(p)} + \frac{19087}{60480} h^7 y^{[7]} \tag{B.11}$$

where $y_{i+1}^{(p)}$ is the right-hand side of (B.5) excluding the remainder term. Similarly, from (B.8) and (B.10)

$$y_{i+1} = y_{i+1}^{(c)} - \frac{863}{60480} h^7 y^{[7]} \tag{B.12}$$

Subtracting (B.12) from (B.11), we obtain in the limit (dropping the subscript i+1)

$$0 = \left[y^{(p)} - y^{(c)}\right] + \frac{19087 + 863}{60480} h^7 y^{[7]} = \left[y^{(p)} - y^{(c)}\right] + \frac{19950}{60480} h^7 y^{[7]}$$

$$h^7 y^{[7]} = \frac{60480}{19950}\left[y^{(c)} - y^{(p)}\right]$$

This approximation for $h^7 y^{[7]}$ permits us to estimate the error terms (B.9) and (B.10). Applying these estimates as corrections, or modifications, to the predicted and corrected values, we obtain, finally, the <u>modified</u> predictor and corrector values:

$$y^{(p,m)} = y^{(p)} + \frac{19087}{60480} h^7 y^{[7]} = y^{(p)} + \frac{19087}{60480} \cdot \frac{60480}{19950}\left[y^{(c)} - y^{(p)}\right]$$

$$y^{(p,m)} = y^{(p)} + \frac{19087}{19950}\left[y^{(c)} - y^{(p)}\right]_{\text{previous step}}$$

$$= y^{(p)} + (0.956741854)\left[y^{(c)} - y^{(p)}\right]_{\text{previous step}}$$

$$y^{(c,m)} = y^{(c)} - \frac{863}{60480} \cdot \frac{60480}{19950}\left[y^{(c)} - y^{(p)}\right] = y^{(c)} - \frac{863}{19950}\left[y^{(c)} - y^{(p)}\right]$$

$$y^{(c,m)} = \left[19087\, y^{(c)} + 863\, y^{(p)}\right] / 19950 \ldots$$

The use of modifiers of this type is often attributed to Hamming[15], although the basic idea goes back at least to Richardson[16].

2.3 Flow Chart

Assume that six previous state vectors Y and right hand sides F are available, either from previous predictor-corrector steps or (at the beginning of an integration) from the Runge-Kutta starting algorithm.

Time	State vector	Right-hand sides
t_1	Y_1	F_1
t_2	Y_2	F_2
t_3	Y_3	F_3
t_4	Y_4	F_4
t_5	Y_5	F_5
t_6	Y_6	F_6

Also required are the intermediate predictor and corrector values at the sixth step, YP_6 and YC_6. Then the following flow chart describes the four operations leading to the values

Time	State vector	Right-hand sides
t_7	Y_7	F_7

at the next time step.

$$YP_7 \longleftarrow Y_6 + \frac{h}{1440} \sum_{i=0}^{5} \alpha_i F_{6-i}$$

$$YPM_7 \longleftarrow YP_7 + \beta \left[YC_6 - YP_6 \right]$$

Evaluate right-hand sides at t_7, using YPM_7. Result is FP_7

$$YC_7 \longleftarrow Y_6 + \frac{h}{1440} \left[\gamma_0 FP_7 + \sum_{i=1}^{5} \gamma_i F_{7-i} \right]$$

$$Y_7 \longleftarrow \left[\lambda \, YC_7 + \mu \, YP_7 \right] / \nu$$

Evaluate right-hand sides using Y_7. Result is F_7. Now the 7th line is complete.

Notation

YP	predicted values
YPM	modified predicted values
YC	corrected values
Y	modified corrected values (final values)
FP	right-hand sides computed using YPM
F	right-hand sides computed using Y (final values)
h	step size

Constants

i	=	0	1	2	3	4	5
α_i	=	4277	-7923	9982	-7298	2877	-475

$$\beta = 0.956741854$$

i	=	0	1	2	3	4	5
δ_i	=	475	1427	-798	482	-173	27

$$\lambda = 19087$$

$$\mu = 863$$

$$\nu = 19950$$

3. Testing of the Predictor-Corrector Algorithm

Assurance that the numerical integration algorithm operates at the required level of accuracy is guaranteed by a series of tests performed both before and after the algorithm was incorporated into CADNIP.

1. To verify the accuracy of the basic predictor-corrector formulas, tests were run using a Runge-Kutta formula, a fourth-order Adams-Bashforth-Moulton procedure, and the presently implemented sixth-order Adams-Bashforth-Moulton method, to generate positions in Keplerian orbits of various eccentricities.

2. The integration method with modifiers was compared with the basic unmodified method, demonstrating the increased accuracy resulting from the use of modifiers. No loss of stability was observed.

3. The truncation error growth as a function of step size was found to conform with theory. Numerical evidence indicated that the growth of round off error was insignificant.

4. Runs were made, using typical satellite orbits, to determine empirically the most appropriate step sizes for the Runge-Kutta and predictor-corrector algorithms.

5. With the predictor-corrector method incorporated operationally into the CADNIP program, test runs were made using actual satellite data:

   i) using Runge-Kutta alone

   ii) using Runge-Kutta only as a starter with the predictor-corrector method being used to generate the orbit.

The results agreed with one another, and both reproduced the satellite motion to within an acceptable level of residuals.

4. Stability of the Method

As a practical matter, the stability of the predictor-modifier-corrector-modifier algorithm has been established empirically by the tests described in the previous section, as well as by the absence of symptoms of instability during the extensive production runs using the CADNIP program.

From a theoretical point of view, the stability of a class of algorithms including the method used in CADNIP has been investigated by Abdel Karim,[17] whose principal result is a theorem giving necessary and sufficient conditions for stability in terms of the positive-definiteness of certain Hermitian forms associated with the system of differential equations. Unlike earlier asymptotic stability theorems which apply only in the limit as $h \rightarrow 0$, the criteria of Abdel Karim are functionally dependent on h and can be used, therefore, to define stability boundaries. It is clear, however, for the particular method being considered here, that values of h large enough to induce instability do not arise in the CADNIP application, where the step size is limited by the relatively stringent propagated truncation error requirement.

Appendix C

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Appendix D

CONTRIBUTORS

The computer program documented in this report was written by Mr. A.S. Bramson of IBM who also formulated many of the numerical techniques employed.

The numerical integration algorithms described in appendix B were developed by Mr. A.R. LeSchack of IBM.

The differential correction procedure was formulated by Professor P. Sconzo of IBM who also provided expert advise and assistance on other aspects of this project.

Mr. J.W. Slowey of the Smithsonian Astrophysical Observatory served as a consultant on the entire project and was instrumental in the development of the atmospheric model and the preliminary adjustment procedure used by the program. In addition, Mr. Slowey was responsible for the investigation of the analytic representation for the geopotential.

Finally, the experience of the following was also drawn upon:

Dr. K.S. Champion, AFCRL
Mr. G.A. Ouellette, IBM
Dr. J.P. Rossoni, IBM

Appendix E

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Related Contracts

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13. ABSTRACT

The Cambridge Atmospheric Density Numerical Integration Program (CADNIP) is a completely automatic computer program capable of determining atmospheric densities from an analysis of satellite observations. The adopted approach consists of a numerical integration procedure combined with a differential correction scheme where discrepancies between computed and observed satellite position and velocity are reconciled by adjusting the assumed atmospheric model, thereby yielding corrected or refined density data.

This report documents the latest version of the program which includes significant refinements and improvements incorporated into CADNIP under this contract.

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14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
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Atmospheric Densities						
Upper Atmosphere						
Model Atmosphere						
Air Drag						
Geopotential						
Harmonics						
Gravitational						
Satellite Orbits						
Satellite Observations						
Orbital Elements						
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